higher education & training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

NATIONAL CERTIFICATE

CONTROL SYSTEMS N6

(8080016)

8 December 2023 (X-paper)
09:00–12:00

REQUIREMENTS: 3-cycle semi-logarithmic graph paper;
graph paper.

This question paper consists of 7 pages, 1 diagram sheet (Nichol's chart), a formula sheet of 2 pages and Laplace transforms of 3 pages.

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DEPARTMENT OF HIGHER EDUCATION AND TRAINING
REPUBLIC OF SOUTH AFRICA

NATIONAL CERTIFICATE
CONTROL SYSTEMS N6
TIME: 3 HOURS
MARKS: 100

INSTRUCTIONS AND INFORMATION

1. Answer all the questions.
2. Read all the questions carefully.
3. Number the answers according to the numbering system used in this question paper.
4. On completion, insert the 3-cycle semi-logarithmic graph paper of the Bode plot and the graph paper of the Root Locus plot into the ANSWER BOOK before handing it in.
5. Write neatly and legibly.

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QUESTION 1

Indicate whether the following statements are TRUE or FALSE by writing only 'True' or 'False' next to the question number (1.1–1.10) in the ANSWER BOOK.

1.1 Total response is that part of the response which approaches zero as time approaches infinity.

1.2 Time period is the time taken by the response to complete one full cycle.

1.3 A steady state error is the difference between the output response and the reference input after all the transient responses have died out.

1.4 Feedback of a system is that range of frequencies over which the system will respond satisfactorily.

1.5 An open-loop control system cannot become unstable because there are no feedback elements to adjust to make the output uncontrollable.

1.6 If the output of a system grows uncontrollably, even if no signal is applied to the system, the system is said to be unstable.

1.7 A closed-loop control system is one in which the control action is independent of the output.

1.8 A step function can be described as a function of time which is periodically changing at a frequency of ω rad/sec.

1.9 Undamped natural frequency describes the condition when two complementary energy-storing components of a system produce an oscillation between them.

1.10 Overshoot refers to the minimum difference between the transient and the steady state solution.

(10 × 1) **[10]**

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QUESTION 2

Determine the total output, C, in terms of all inputs, with the aid of the block diagram algebra reduction in FIGURE 1.

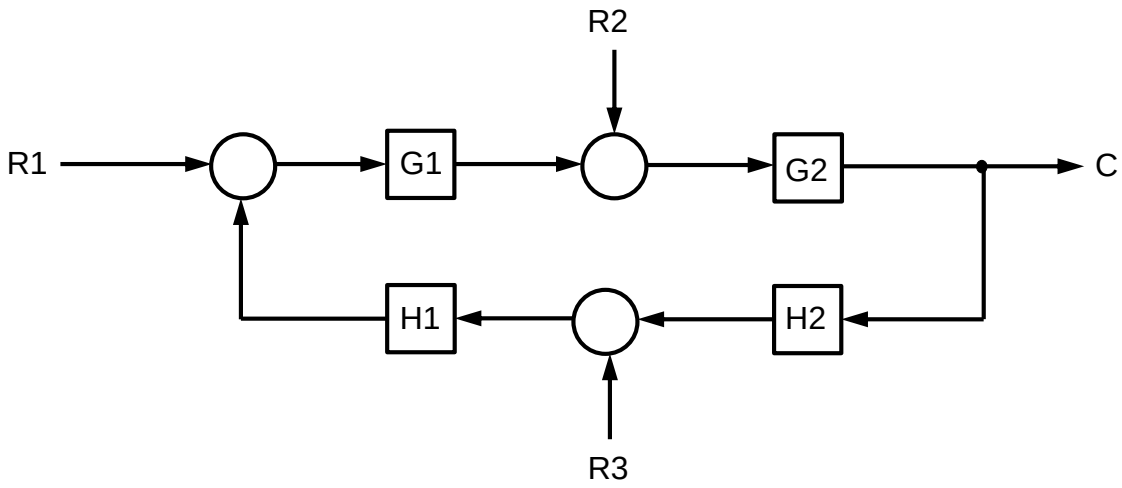


FIGURE 1 **[10]**

QUESTION 3

The transfer function of the forward path and the feedback path of a simple closed loop control system with negative feedback is given as:

$$G(s) = \frac{38{,}4}{(s+1)(s+5)} \qquad H(s) = \frac{12(s+1)}{s(s+10)}$$

Plot, on the same 3-cycle semi-logarithmic graph paper, the asymptotic gain and phase loci for the system. Do not sum the loci on the graph. **[10]**

QUESTION 4

DIAGRAM SHEET 1 (attached) illustrates a closed-loop gain versus phase plot on a Nichols chart.

4.1 Use the Nichols chart to determine the following:

4.1.1 The gain margin (1)

4.1.2 The phase margin (1)

4.1.3 The gain crossover frequency (1)

4.1.4 The phase crossover frequency (1)

4.1.5 The un-damped natural resonant frequency (1)

4.1.6 The peak frequency response (1)

4.1.7 The peak magnitude and phase (2)

4.1.8 The closed-loop bandwidth (1)

4.2 State whether the system is stable or unstable. (1)

[10]

☆

QUESTION 5

The TABLE below shows the loci values of a closed-loop transfer function, as K varies from 0 to infinity.

K_0	0	1.5	4	12	15	50	∞
S_1	0	- 1	-1.3 + j1	- 3 + j2	- 4.4 + j1.4	- 5	- 3
S_2	- 1.8	- 1	-1.3 – j1	- 3 – j2	- 4.4 – j1.4	- 5	- 67493

HINT: The break-away point occurs at – 1.

5.1 Construct the root locus for this system on the graph paper. (6)

5.2 Determine the following from the root locus:

5.2.1 The damped resonant frequency. (1)

5.2.2 The natural frequency. (1)

☆ 5.2.3 Calculate the damping ratio. (2)

[10]

QUESTION 6

6.1 Use Laplace transforms to convert the following functions to the ‘s’ domain.

6.1.1 $f(x) = 12cos3t$ (1)

6.1.2 $f(x) = 12(t)$ (1)

6.1.3 $F(t) = 30(1 - e^{\frac{-t}{3}})$ (2)

☆

6.2 Use Laplace transforms to convert the following functions to the 't' domain.

6.2.1 $F(s) = \frac{24}{(s+4)^2}$ (1)

6.2.2 $F(s) = \frac{15\omega}{(s^2+\omega^2)(s+5)}$ (2)

6.2.3 $F(s) = \frac{42}{(4s+1)(2s+1)}$ (3)

[10]

QUESTION 7

7.1 FIGURE 2 shows a diagram of an operational amplifier.

Identify the terminals labelled A, B and C.

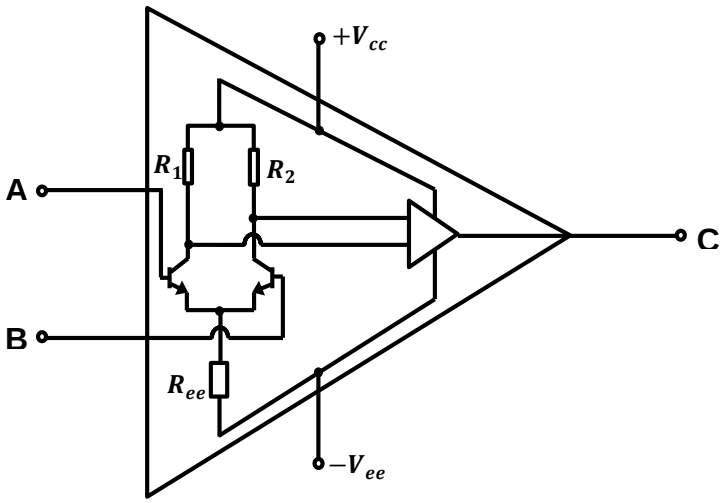


FIGURE 2 (3)

7.2 State FOUR typical uses of operational amplifiers. (4)

7.3 Name THREE components of an optical relay. (3)

[10]

QUESTION 8

8.1 Name the TWO types of semi-rotary actuators or motors. (2)

8.2 Explain the term, *derivative control,* relating to process controllers which are used in hydraulic systems. (1)

8.3 Draw a neat, labelled diagram of a basic hydraulic system, reciprocating machine tool. (7)

[10]

QUESTION 9

9.1 Name the THREE cascade-connected control terms of a pneumatic controller. (3)

9.2 Define the term, *integral control action*. (2)

9.3 State the TWO disadvantages of using an on-off control. (2)

9.4 State THREE disadvantages of using pneumatic systems. (3)

[10]

QUESTION 10

10.1 State what Lissajous figures are usually used for. (2)

10.2 Name the most common user error when using an oscilloscope. (1)

10.3 A sinusoidal response causes a pulse to have an effect on an R-C circuit.

Draw the waveforms produced in response to a pulse when:

10.3.1 R.C << $\mathbb{T}$

10.3.2 R.C = $\mathbb{T}$

10.3.3 R.C >> $\mathbb{T}$

(3 × 1) (3)

10.4 Explain how a saw-tooth wave can be generated. (2)

10.5 A pulse generator produces a pulse train with a mark to space ratio of 1µs to 4µs.

Calculate the pulse repetition frequency of the waveform. (2)

HINT: $p.r.f = \frac{1}{Period}$

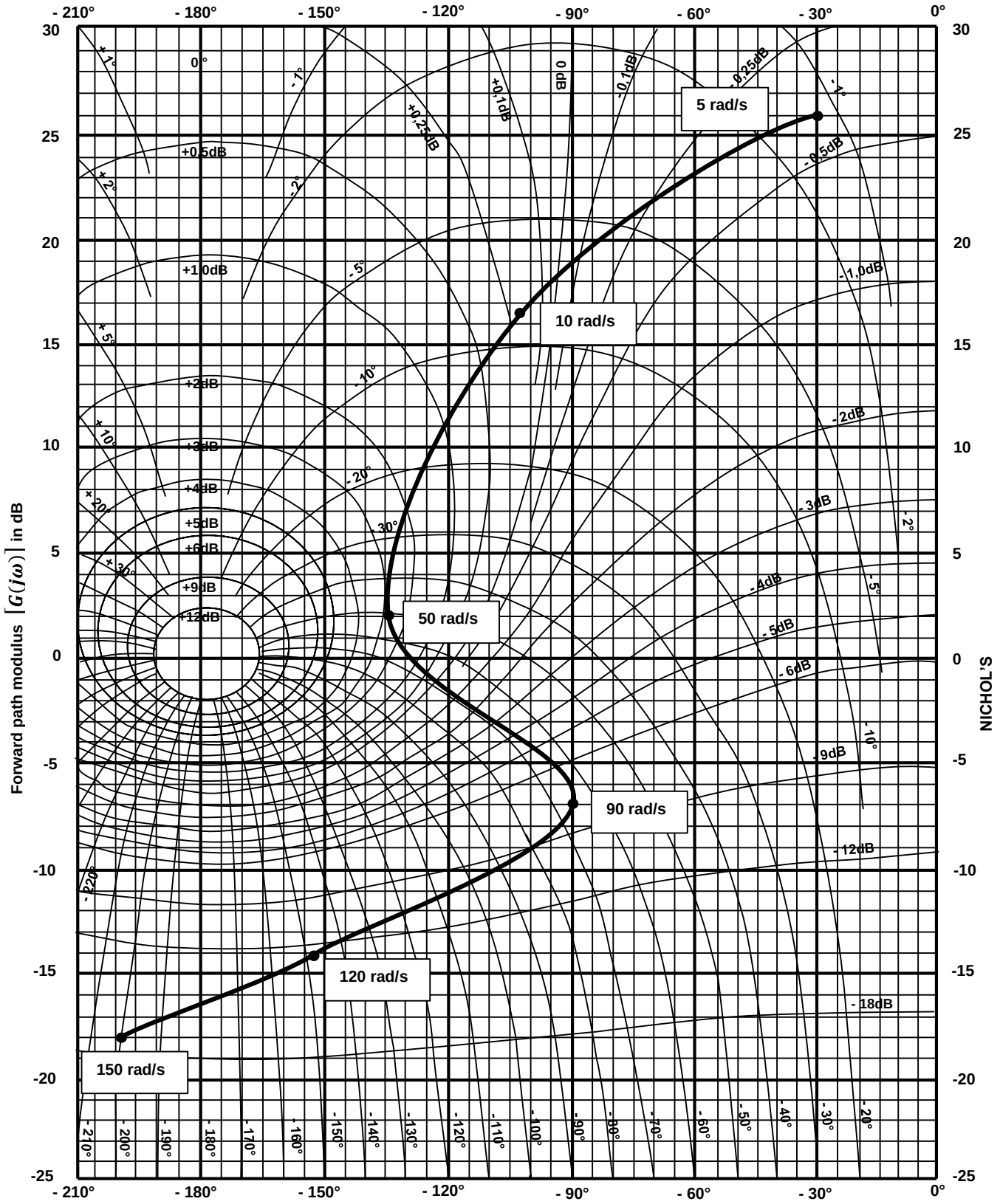
[10]

TOTAL: 100

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DIAGRAM SHEET 1



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FORMULA SHEET

$\omega_d = \omega_n\sqrt{1-\zeta^2} = 2\pi f$ $\qquad t_p = \frac{1}{f}$

Number of oscillations $\frac{t_s}{t_p}$ or / of $\frac{2\sqrt{1-\zeta^2}}{\pi.\zeta}$

Damping coefficient $(\alpha) = \zeta . \omega_n = \frac{1}{\pi}\tau$

Overshoot $= e^{\frac{-\zeta\pi N}{\sqrt{1-\zeta^2}}}$

$\Psi = tan^{-1}\left[\frac{\sqrt{1-\zeta^2}}{-\zeta}\right] + \pi\, rad$

Amplitude $= \varphi\left[1 + e^{\frac{-\zeta\pi N}{\sqrt{1-\zeta^2}}}\right]$

$\omega_n = \sqrt{\frac{Ko}{\tau}}$ $\qquad \tau + \frac{t_s}{4} = \frac{1}{\zeta . \omega_n}$

$\omega_p = \omega_n\sqrt{1-2\zeta^2}$

$\omega_b = \omega_n\sqrt{1-2\zeta^2+\sqrt{2-4\zeta^2+4\zeta^4}}$

$\frac{C}{R} = \frac{G}{1 \pm GH}$ $\qquad Sc = \frac{\sum P - \sum Z}{NP-NZ}$

$\zeta = COS\,\varphi$ $\qquad \Psi = \frac{(2Ko+1)180°}{NP-NZ}$

$Ko = \frac{\Delta P_1 . \Delta P_{2....}}{\Delta Z_1 . \Delta Z_{2....}}$

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AMPLIFIERS

$$\mathrm{Vo} = -V_i \frac{\mathrm{R}_f}{\mathrm{R}_1}$$

$$\mathrm{Vo} = V_i \left[1 + \frac{\mathrm{R}_f}{\mathrm{R}_1}\right]$$

$$\mathrm{Vo} = -R_f \left[\frac{\mathrm{V}_1}{\mathrm{R}_1} + \frac{V_2}{R_2} + \cdots \frac{V_N}{R_N}\right]$$

$$\mathrm{Vo} = -\frac{1}{RC} \int V_i(t)dt + V_c$$

$$\mathrm{Vo} = \frac{R_f}{R_1}[V_2 - V_1]$$

$$\mathrm{Vo} = -RC \frac{dV_i(t)}{dt}$$

$$\mathrm{i}_e = \frac{V_e}{R_e}$$

$$R_c = \frac{V_c}{i_c}$$

$$\mathrm{gm}R_L = \frac{hfe}{hie}.R_L$$

$$\mathrm{t} = \frac{1}{f}$$

BODE AND NICHOLS CHARTS

$$Gain = 20\log\left[\frac{output}{input}\right] db$$

$$Gain = 20\log\left[\frac{output}{input}\right] db$$

$$Phase = \mathrm{Sin}^{-1}\left[\frac{phase\ shift}{input}\right] - 180°$$

$$Phase = \mathrm{Sin}^{-1}\left[\frac{phase\ shift}{input}\right] - 180°$$

$$\tau = R.C$$

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LAPLACE TRANSFORM TABLE

No	F(s)	f(t)
1.	1	$\delta(t)$
2.	$\frac{A}{s}$	$A(t)$ $\{0 \ \ t < 0\}$ $\{A \ \ t \geq 0\}$
3.	$\frac{1}{s}$	$U(t)$ $\{0 \ \ t < 0\}$ $\{1 \ \ t \geq 0\}$
4.	$\frac{A}{s^2}$	At
5.	$\frac{2A}{s^3}$	At^2
6.	$\frac{A_\omega}{s^2 + \omega^2}$	$A \sin \omega t$
7.	$\frac{A_s}{s^2 + \omega^2}$	$A \cos \omega t$
8(a).	$\frac{A}{\tau s + 1}$	$\frac{A}{\tau} e^{\frac{-t}{\tau}}$
8(b).	$\frac{A}{s + a}$	Ae^{-at}
9(a).	$\frac{A}{(\tau_1 s + 1)(\tau_2 s + 1)}$	$\frac{A}{\tau_1 - \tau_2} \left[e^{\frac{-t}{\tau_1} - \frac{-t}{\tau_2}}\right]$
9(b).	$\frac{A}{(s + a)(s + b)}$	$\frac{A}{(b - a)} \ [e^{-at} - e^{-bt}]$
10(a).	$\frac{A}{(\tau_s + 1)^2}$	$\frac{At}{\tau^2} \left[e^{\frac{-t}{\tau}}\right]$
10(b).	$\frac{A}{(s + a)^2}$	Ate^{-at}
11.	$\frac{A{\omega_n}^2}{s^2 + 2\zeta\omega_n s + {\omega_n}^2}$	$\frac{A\omega_n e^{-\zeta\omega_n t}}{\sqrt{1 - \zeta^2}} \sin(\omega_n \sqrt{1 - \zeta^2 t}$
12(a).	$\frac{A}{s(\tau s + 1)}$	$A(1 - e^{\frac{-t}{\tau}})$
12(b).	$\frac{A}{s(s + a)}$	$\frac{A}{a}(1 - e^{-at})$
13(a).	$\frac{A}{s^2(\tau s + 1)}$	$A\tau \left[e^{\frac{-t}{\tau}} + \frac{t}{\tau} - 1\right]$
13(b).	$\frac{A}{s^2(s + a)}$	$\frac{A}{a^2} (e^{-at} + at - 1)$

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14(a).	$\frac{A\omega}{(s^2+\omega^2)(\tau s+1)}$	$\frac{A\omega\tau}{1+\omega^2\tau^2}e^{\frac{-t}{\tau}}+\frac{A}{\sqrt{1+\omega^2\tau^2}}\sin(\omega t-\Psi)$ *where* $\Psi=\tan^{-1}\omega\tau\ (0<\Psi<\pi)$
14(b).	$\frac{A\omega}{(s^2+\omega^2)(s+a)}$	$\frac{A\omega e^{-at}}{(\omega^2+a^2)}+\frac{A}{\sqrt{\omega^2+a^2}}\sin(\omega t-\Psi)$ *where* $\Psi=\tan^{-1}\frac{\omega}{a}\ (0<\Psi<\pi)$
15(a).	$\frac{A}{s(\tau_1 s+a)(\tau_2 s+1)}$	$A\left[1+\frac{\tau_1 e^{\frac{-t}{\tau_1}}-\tau_2 e^{\frac{-t}{\tau_2}}}{\tau_1-\tau_2}\right]$
15(b).	$\frac{A}{s(s+a)(s+b)}$	$\frac{A}{ab}\left[1+\frac{ae^{-bt}-be^{-at}}{b-a}\right]$
16(a).	$\frac{A}{s(\tau+a)^2}$	$A\left[1-\frac{(\tau+t)}{\tau}e^{\frac{-t}{\tau}}\right]$
16(b).	$\frac{A}{s(s+1)^2}$	$\frac{A}{a^2}\left[1-(1+at)e^{-at}\right]$
17.	$\frac{A{\omega_n}^2}{s(s^2+2\zeta\omega_n s+{\omega_n}^2)}$	$A\left[1+\frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}\sin(\omega_n t\sqrt{1-\zeta^2}-\Psi\right]$ *where* $\Psi=tan^{-1}\frac{\sqrt{1-\zeta^2}}{-\zeta}\ (0<\Psi<\pi)$
18(a).	$\frac{A}{s^2(\tau_1 s+1)(\tau_2 s+1)}$	$A\left[t-\tau_1-\tau_2-\frac{{\tau_2}^2 e^{\frac{-t}{\tau_2}}-{\tau_1}^2 e^{\frac{-t}{\tau_1}}}{\tau_1-\tau_2}\right]$
18(b).	$\frac{A}{s^2(s+a)(s+b)}$	$\frac{A}{ab}\left[t-\frac{a+b}{ab}-\frac{\frac{b}{a}e^{-bt}-\frac{a}{b}e^{-at}}{b-a}\right]$
19(a).	$\frac{A}{s^2(\tau s+1)^2}$	$A\left[t-2\tau+(t+2\tau)e^{\frac{-t}{\tau}}\right]$
19(b).	$\frac{A}{s^2(s+a)^2}$	$\frac{A}{a^2}\left[t-\frac{2}{a}+\left(t+\frac{2}{a}\right)e^{-at}\right]$
20.	$\frac{A{\omega_n}^2}{s^2(s^2+2\zeta\omega_n s+{\omega_n}^2)}$	$A\left[\tau-\frac{2\zeta}{\omega_n}+\frac{e^{-\zeta\omega_n t}}{\omega_n\sqrt{1-\zeta^2}}\sin(\omega_n t\sqrt{1-\zeta^2}-\Psi\right]$ *where* $\Psi=2tan^{-1}\frac{\sqrt{1-\zeta^2}}{-\zeta}\ (0<\Psi<\pi)$

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21(a).	$\frac{A\omega}{(s^2+\omega^2)(\tau_1 s+1)(\tau_2 s+1)}$	$A\left[\frac{{\tau_1}^2\omega e^{\frac{-t}{\tau_1}}}{(\tau_1-\tau_2)(1+\omega^2{\tau_1}^2)}+\frac{{\tau_2}^2\omega e^{\frac{-t}{\tau_2}}}{(\tau_1-\tau_2)(1+\omega^2{\tau_2}^2)}+\frac{\sin(\omega t-\Psi)}{(1+\omega^2\tau^2)(1+\omega^2{\tau_2}^2)^{\frac{1}{2}}}\right]$ *where* $\Psi = tan^{-1}\,\omega\tau_1 + tan^{-1}\omega\tau_2$
21(b).	$\frac{A\omega}{(s^2+\omega^2)(s+a)(s+b)}$	$A\left[\frac{\omega e^{-at}}{(b-a)(\omega^2+a^2)}+\frac{\omega e^{-bt}}{(a-b)(\omega^2+b^2)}+\frac{\sin(\omega t-\Psi)}{(\omega^2+a^2)(\omega^2+b^2)^{\frac{1}{2}}}\right]$ *where* $\Psi = tan^{-1}\,\frac{\omega(a+b)}{ab-\omega^2}\ (0<\Psi<\pi)$
22(a).	$\frac{A\omega}{(s^2+\omega^2)(\tau s+1)^2}$	$\frac{A}{1+\omega^2\tau^2}\left[\frac{\omega t+2\omega\tau}{1+\omega^2\tau^2}\,e^{\frac{-t}{\tau}}+\sin(\omega t-\Psi\right]$ *where* $\Psi = 2tan^{-1}\,\omega\tau$
22(b).	$\frac{A\omega}{(s^2+\omega^2)(s+a)^2}$	$\frac{A}{\omega^2+a^2}\left[\frac{a\omega(at+2)e^{-at}}{\omega^2+a^2}+\sin(\omega t-\Psi\right]$
23.	$\frac{A\omega{\omega_n}^2}{(s^2+\omega^2)(s^2+2\zeta\omega_n s+{\omega_n}^2)}$	$\frac{A{\omega_n}^2}{[({\omega_n}^2-\omega^2)^2+4\zeta^2\omega^2{\omega_n}^2]^{\frac{1}{2}}}\left[\sin(\omega t-\Psi)+\frac{\omega e^{-\zeta\omega_n t}\sin(\omega_n\sqrt{1-\zeta^2}\,t-\Psi_2)}{\omega_n\sqrt{1-\zeta^2}}\right]$ *where* $\Psi_1 = tan^{-1}\left[\frac{2\zeta\omega\omega_n}{{\omega_n}^2+\omega^2}\right]\ (0<\Psi_1<\pi)$ and $\Psi_2 = tan^{-1}-\frac{2\zeta{\omega_n}^2\sqrt{1-\zeta^2}}{\omega^2-{\omega_n}^2(1-2\zeta^2)}\ (0<\Psi_2<\pi)$

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