higher education & training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

MARKING GUIDELINE

NATIONAL CERTIFICATE

MATHEMATICS N5

17 NOVEMBER 2022

This marking guideline consists of 10 pages.

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QUESTION 1

1.1 $y = \lim_{x\to\infty} \frac{\ln[\ln(x^2+1)]}{x}$ $\frac{\infty}{\infty}$

$= \lim_{x\to\infty} \frac{2x}{\ln(x^2+1)}$ ✓ $\frac{\infty}{\infty}$

$= \lim_{x\to\infty} \frac{x^2+1}{x}$ ✓ $\frac{\infty}{\infty}$

$= \lim_{x\to\infty} \frac{2x}{1}$ ✓

$= \infty$✓ (4)

1.2 $y = \lim_{x\to 0} \frac{x^3}{e^x - x - \frac{1}{2}x^2 - 1}$ $\frac{0}{0}$

$= \lim_{x\to 0} \frac{3x^2}{e^x - 1 - x}$ ✓ $\frac{0}{0}$

$= \lim_{x\to 0} \frac{6x}{e^x - 1}$ ✓ $\frac{0}{0}$

$= \lim_{x\to 0} \frac{6}{e^x}$ ✓

$= 6$✓

(4)
[8]

QUESTION 2

2.1 $y = arc\tan x$
$\tan y = x$
$\sec^2 y \frac{dy}{dx} = 1$ ✓
$\frac{dy}{dx} = \frac{1}{\sec^2 x}$ ✓
$= \frac{1}{1+\tan^2 x}$ ✓
$= \frac{1}{1+x^2}$ (3)

2.2 $y = \pi^{\log_\pi \cot x}$

$y = \cot x$ ✓

$= \dfrac{\cos x}{\sin x}$

$\dfrac{dy}{dx} = \dfrac{\sin x \times -\sin x ✓ - \cos x \times \cos x ✓}{\sin^2 x}$

$= \dfrac{-1}{\sin^2 x}$ ✓

$= -\operatorname{cosec}^2 x$ (4)

2.3 2.3.1 $y = \tan(x^3 \ln^2 x)$

$\dfrac{dy}{dx} = \sec^2(x^3 \ln^2 x) ✓ \times \left(3x^2 \times \ln^2 x ✓ + x^3 \times 2 \ln x \times \dfrac{1}{x} ✓\right)$ (3)

2.3.2 $y = \dfrac{\sqrt{\ln(2x+3)}}{4\sqrt{3x-1}}$

$$\frac{dy}{dx} = \frac{4\sqrt{3x-1} \times \frac{1}{2} \times [\ln(2x+3)]^{-\frac{1}{2}} \times \frac{1}{2x+3} \times 2 ✓✓ - \left[\sqrt{\ln(2x+3)} \times 4 \times \frac{1}{2}(3x-1)^{-\frac{1}{2}} \times 3\right] ✓✓}{\left(4\sqrt{3x-1}\right)^2}$$ (4)

2.4 $y = (1 + \sin^{-1} x)^{\ln \cos x}$

$\ln y = \ln \cos x \ln(1 + \sin^{-1} x)$

$\dfrac{1}{y}\dfrac{dy}{dx} ✓ = \ln \cos x \times \dfrac{1}{1 + \sin^{-1} x} \times \dfrac{1}{\sqrt{1-x^2}} ✓ + \ln(1 + \sin^{-1} x) \times \dfrac{1}{\cos x} \times -\sin x ✓$

$\dfrac{dy}{dx} = y\left[\dfrac{\ln \cos x}{(1 + \sin^{-1} x)\left(\sqrt{1-x^2}\right)} - \tan x . \ln(1 + \sin^{-1} x)\right] ✓$

$= (1 + \sin^{-1} x)^{\ln \cos x}\left[\dfrac{\ln \cos x}{(1 + \sin^{-1} x)\left(\sqrt{1-x^2}\right)} - \tan x . \ln(1 + \sin^{-1} x)\right]$ (4)

2.5 $\dfrac{2x-3y}{x+y}=2x+y^2$

$2x-3y=(x+y)(2x+y^2)$

$2x-3y=2x^2+2xy+xy^2+y^3$

$2-3\dfrac{dy}{dx}\checkmark=4x+2y+2x\dfrac{dy}{dx}+y^2+2xy\dfrac{dy}{dx}+3y^2\dfrac{dy}{dx}\checkmark\checkmark$

$3y^2\dfrac{dy}{dx}+2xy\dfrac{dy}{dx}+2x\dfrac{dy}{dx}+3\dfrac{dy}{dx}=2-4x-2y-y^2$

$(3y^2+2xy+2x+3)\dfrac{dy}{dx}=2-4x-2y-y^2\checkmark$

$\dfrac{dy}{dx}=\dfrac{2-4x-2y-y^2}{3y^2+2xy+2x+3}\checkmark$ (5)

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QUESTION 3

3.1 3.1.1 $f(x)=3x^3+5x^2-7x-5$

$f'(x)=9x^2+10x-7=0$

$=\dfrac{-(10)\pm\sqrt{(10)^2-4(9)(-7)}}{2(9)}$

$x=-1{,}598 \quad or \quad x=0{,}487\checkmark$

$y=6{,}712 \quad y=-6{,}877\checkmark$

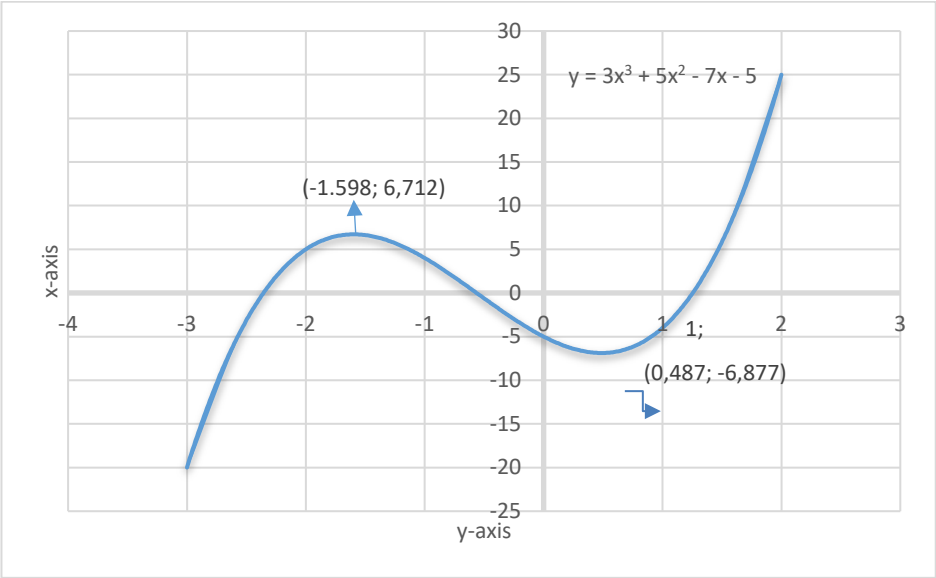
Thus, the coordinates of the turning points are $(-1{,}598;\ 6{,}712)$ and $(0{,}487;\ -6{,}877)\checkmark$ (3)

3.1.2

x	-3	-2	-1	0	1	2
y	-20	5	4	-5	-4	25

1 mark each for any 3 correct answers✓✓✓ (3)

3.1.3



✓ = 1 mark for the shape
✓ = 1 mark for the indication of the turning points on the graph (2)

3.1.4 $Let\ x_0 = -0{,}5$

$f(-0{,}5) = -0{,}625$

$f'(-0{,}5) = -9{,}75$✓

$x_1 = -0{,}5 - \frac{-0{,}625}{-9{,}75}$✓

$= -0{,}564$✓ (3)

3.2 Because of similar triangles we have,

$\frac{r}{h} = \frac{10}{30}$

$\therefore\ r = \frac{h}{3}$✓

$V = \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi\left(\frac{h}{3}\right)^2 h$

$= \frac{\pi h^3}{27}$✓

$\frac{dV}{dt} = \frac{\pi h^2}{9}\frac{dh}{dt}$✓

$10cm^3/s = \frac{\pi(4cm)^2}{9}\frac{dh}{dt}$✓

$\frac{dh}{dt} = \frac{90}{16\pi}cm/s$✓

$= 1{,}790cm/s$ (5)

[15]

QUESTION 4

4.1 4.1.1

$$\int \frac{\sin\left(\frac{1}{x}\right)}{x^2}\,dx$$

$$let\ u = \frac{1}{x}$$

$$du = -\frac{1}{x^2}\,dx\checkmark$$

$$-du = \frac{1}{x^2}\,dx$$

$$= -\int \sin u\ du\checkmark$$

$$= \cos u + C$$

$$= \cos\left(\frac{1}{x}\right) + C\checkmark$$

ALTERNATIVE

$$I = \int \frac{\sin\left(\frac{1}{x}\right)}{x^2}\,dx$$

$$\frac{d}{dx}\cos\left(\frac{1}{x}\right)\checkmark = -sin\left(\frac{1}{x}\right) \times \frac{-1}{x^2}\checkmark$$

$$= \frac{\sin\left(\frac{1}{x}\right)}{x^2}$$

$$I = \int \frac{\sin\left(\frac{1}{x}\right)}{x^2}dx = \cos\left(\frac{1}{x}\right) + C\checkmark$$

(3)

4.1.2

$$\int \frac{2x^3 - 11x^2 - 9x + 10}{2x - 1}\,dx$$

$$= \int \left(x^2 - 5x - 7 + \frac{3}{2x-1}\right)dx\ \checkmark$$

$$= \frac{1}{3}x^3\checkmark - \frac{5}{2}x^2\checkmark - 7x\checkmark + \frac{3}{2}\ln(2x-1)\checkmark + C$$

(5)

4.1.3

$$\int x^2 e^{5x}\,dx$$

$$let\ f(x) = x^2 \qquad g'(x) = e^{5x}\,dx$$

$$f'(x) = 2x \qquad g(x) = \frac{1}{5}e^{5x}\ \checkmark$$

$$\int x^2 e^{5x}\,dx = \frac{x^2 e^{5x}}{5}\checkmark - \frac{2}{5}\int xe^{5x}\,dx\checkmark$$

$$let\ f(x) = x \qquad g'(x) = e^{5x}\,dx$$

$$f'(x) = 1 \qquad g(x) = \frac{1}{5}e^{5x}\ \checkmark$$

$$= \frac{x^2 e^{5x}}{5} - \frac{2}{5}\left(\frac{xe^{5x}}{5}\checkmark - \frac{1}{5}\int e^{5x}\,dx\checkmark\right)$$

$$= \frac{x^2 e^{5x}}{5} - \frac{2xe^{5x}}{25} + \frac{2}{125}e^{5x}\checkmark + C$$

(7)

4.1.4 $$y = \frac{\sin^3 x}{\cos^3 x}$$

$$= \int \tan^3 x\, dx \checkmark$$

$$= \int \tan^2 x \tan x\, dx$$

$$= \int (\tan^2 x - 1)\tan x\, dx \checkmark$$

$$= \int \tan^2 x \tan x\, dx - \int \tan x\, dx$$

$$= -\frac{1}{2}\tan^2 x \checkmark - \ln(\sec x)\checkmark + C$$ (4)

4.2 $$\int \frac{2x-3}{x^2-4x}dx$$

$$\frac{2x-3}{x(x-4)} = \frac{A}{x} + \frac{B}{x-4}$$

$$2x - 3 = A(x-4) + Bx\checkmark$$

$let\ x = 4; \qquad B = \frac{5}{4}\checkmark$ or $x = 0; \qquad A = \frac{3}{4}\checkmark$

$$\int \left(\frac{\frac{3}{4}}{x} + \frac{\frac{5}{4}}{x-4}\right)dx$$

$$= \frac{3}{4}\ln x\checkmark + \frac{5}{4}\ln(x-4)\checkmark + C$$ (5)

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QUESTION 5

5.1

$$\int_0^1 f'(x)\,[f(x)]^2\,dx$$

$$let\ u = f(x)$$
$$du = f'(x) \quad \checkmark$$
$$u = f(0) = 1 \quad and \quad u = f(1) = 3$$

$$= \int_1^3 u^2 du \ \checkmark$$

$$= \left[\frac{1}{3}u^3\right]_1^3 \ \checkmark$$

$$= \frac{1}{3} \times 3^3 - \frac{1}{3} \times 1^3$$

$$= \frac{26}{3} \ \checkmark$$

(4)

5.2 5.2.1 $x^2 - 2x + 1 = 5x - x^2$

$2x^2 - 7x + 1 = 0$

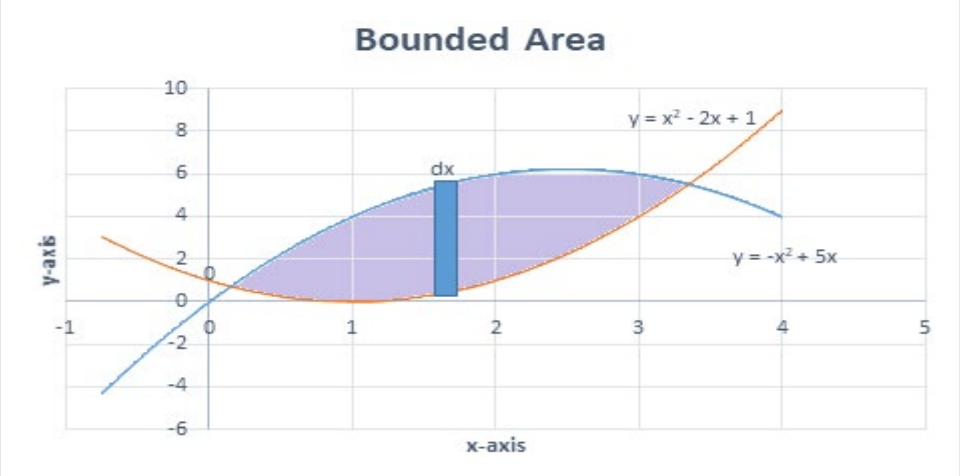
$$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(2)(1)}}{2(2)}$$

$x = 0{,}149 \ \ or \ \ x = 3{,}351\checkmark$

$y = 0{,}723 \quad y = 5{,}526$

Thus, the coordinates of the points of intersection are $(0{,}149\ ; 0{,}723)$ and $(3{,}351;\ 5{,}526).\checkmark$ (2)

5.2.2



✓ = 1 mark for indication of enclosed area
✓ = 1 mark for indication of the vertical or horizontal strip (2)

5.2.3

$$A = \int_a^b (y_1 - y_2)\,dx$$

$$= \int_{0,149}^{3,351} [(5x - x^2) - (x^2 - 2x + 1)]\,dx$$

$$= \int_{0.149}^{3,351} (-2x^2 + 7x + 1)dx \checkmark$$

$$= \left[-\frac{2}{3}x^3 + \frac{7}{2}x^2 - x\right]_{0,149}^{3,351} \checkmark$$

$$= 10,939 units^2 \quad \checkmark$$ (3)

5.2.4

$$V = \pi \int_a^b ({y_1}^2 - {y_2}^2)\,dx$$

$$= \pi \int_{0,149}^{3,351} [(5x - x^2)^2 - (x^2 - 2x + 1)^2]\,dx$$

$$= \int_{0,149}^{3,351} (-6x^3 + 19x^2 + 4x - 1)dx \checkmark\checkmark$$

$$= \left[-\frac{3}{2}x^4 + \frac{19}{3}x^3 + x^2 - x\right]_{0,149}^{3,351} \checkmark$$

$$= 214,780 units^3 \checkmark$$ (4)

5.3

$dI = x^2\,dm$

$= x^2 \rho b\,dx \checkmark$

$= 1bx^2\,dx \checkmark$

$$I = \int_0^a bx^2\,dx \checkmark$$

$$= \left[\frac{b}{3}x^3\right]_0^a \checkmark$$

$$= \frac{ba^3}{3} \checkmark$$ (5)

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QUESTION 6

6.1 $(x-2)(y+1)\frac{dy}{dx} - \log_5(x-2) = 0$

$(y+1)\,dy = \frac{\log_5(x-2)}{x-2}dx$

$(y+1)\,dy = \frac{1}{\ln 5}\frac{\ln(x-2)}{x-2}dx$ ✓

$\frac{1}{2}y^2 + y$✓ $= \frac{1}{2\ln 5}\ln^2(x-2) + C$ ✓ (3)

6.2 $\frac{d^2y}{dx^2} = 4 - 6x - 21x^2$

$\frac{dy}{dx} = A + 4x - 3x^2 - 7x^3$✓

$2 = A + 4(1) - 3(1)^2 - 7(1)^3$

$A = 8$✓

$\frac{dy}{dx} = 8 + 4x - 3x^2 - 7x^3$✓

$y = B + 8x + 2x^2 - x^3 - \frac{7}{4}x^4$✓

$10 = B + 8(1) + 2(1)^2 - (1)^3 - \frac{7}{4}(1)^4$

$B = \frac{11}{4}$✓

$y = \frac{11}{4} + 8x + 2x^2 - x^3 - \frac{7}{4}x^4$✓ (6)

[9]

TOTAL: 100

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