N5

Fluid Mechanics

Lecturer Guide

Peet du Toit

Additional resource material for this title includes:

- Electronic Lecturer Guide
- PowerPoint presentation
- Past exam papers

Scan the QR code below or visit this link: futman.pub/N5FluidMechLG

SCAN ME

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Contents

Lecturer Guidance

1. General aims

To equip students with the necessary knowledge, principles and practices in the mechanical engineering field and to solve problems.

2. Specific aims

2.1 The students should obtain an understanding of Fluid Mechanics as it is applied in the industry.
2.2 To enable the students to conceptualise and deal with specific and complex issues and problems in the field of engineering.
2.3 To provide a basic understanding with reference to systems and components used in the engineering industry.

3. Pre-requisites

Students must meet at least one of the following requirements:
3.1 National N4 certificate with Engineering Science or Machines and Properties of Metals N4
3.2 Relevant skills programme registered on the NQF with at least 120 credits.

4. Duration

Full-time: 7.5 hours per week. This instructional offering may also be offered on a part-time basis.

5. Evaluation

5.1 Evaluation is conducted continuously by means of two formal tests at College level. Learner must obtain a minimum ICASS mark of at least 40% in order to qualify to write the final examination, and a mark will be calculated together in a ratio of 40:60 to derive the promotion mark. The learner must obtain at least 40% in the final examination.

The promotion mark will be calculated as follows:

Promotion Mark = **40%** of (ICASS mark) **+ 60%** of (Exam mark)

5.2 The examination in Fluid Mechanics N5 (Engineering Studies – Report 191) will be conducted as follows:
Topics 1–11: 100 marks
Duration: 3 hours
Closed book:

- Formula sheet attached to the question paper
- Scientific calculators allowed
- No programmable calculators allowed
- No references allowed
- No external examination papers or memoranda allowed
- No cellphones allowed
- No smart watches allowed.

5.3 **Weighting**
The following weights are awarded to each category:

Knowledge and understanding	Application	Analysing/synthesis and evaluation
10–15	65–75	15–20

6. Learning content

THEORETICAL BACKGROUND
It is essential that this subject should be explained and evaluated within the context of technical skills and the simulation of a practical environment.

7. Mark allocation and weighted values of modules

Mark allocation in the examination as an indication of the weighting of the different modules:

Module	Description	Weighted value (%)
	Section 1: Hydrostatics	
1	Properties of fluids	8
2	Pressure systems	6
3	Simple hydraulics systems	12
4	Hydrostatic forces on submerged areas	9
5	Buoyancy and stability of floating and immersed bodies	9

Module	Description	Weighted value (%)
Section 2: Hydrodynamics		
6	Fluid in motion	12
7	Flow measurement using instruments	10
8	Pipeline systems	10
9	Flow through orifices	8
10	Conservation of fluid momentum	8
11	Reciprocating pumps	8
Total		**100**

8. Work schedule

Week	Topic	Content	Exercises	Hours
Section 1: Hydrostatics				
1	**Module 1** Properties of fluids	1.1 Definition of a fluid 1.2 Physical properties of fluids	Exercise 1.1 Exercise 1.2 Exercise 1.3 Exercise 1.4 Summative assessment	± 5 hours
1–2	**Module 2** Pressure systems	2.1 Static pressure 2.2 Types of pressure 2.3 Pressure calculations on manometers as pressure measuring devices	Exercise 2.1 Exercise 2.2 Summative assessment	± 10 hours
3	**Module 3** Simple hydraulic systems	3.1 Actuating cylinders 3.2 Viscous resistance in rotating shafts or pistons	Exercise 3.1 Exercise 3.2 Exercise 3.3 Exercise 3.4 Exercise 3.5 Summative assessment	± 10 hours

Week	Topic	Content	Exercises	Hours
4–5	**Module 4** Hydrostatic forces on submerged areas	4.1 Hydrostatic force on flat surfaces and the centre of pressure 4.2 Hydrostatic forces on rectangular and circular tanks containing surface pressure on two liquids 4.3 Hydrostatic forces on plane submerged surfaces	Exercise 4.1 Exercise 4.2 Exercise 4.3 Summative assessment	± 10 hours
5	**Module 5** Buoyancy and stability of floating and immersed bodies	5.1 Archimedes' principle 5.2 Buoyancy	Exercise 5.1 Exercise 5.2 Exercise 5.3 Summative assessment	± 10 hours
Section 2: Hydrodynamics				
6	**Module 6** Fluid in motion	6.1 The nature of fluids and types of flow 6.2 Fluid in motion calculations	Exercise 6.1 Exercise 6.2 Summative assessment	± 10 hours
7	**Module 7** Flow measurement using instruments	7.1 Flow measurement	Exercise 7.1 Exercise 7.2 Summative assessment	± 10 hours
7–8	**Module 8** Pipeline systems	8.1 Losses in pipelines	Exercise 8.1 Exercise 8.2 Summative assessment	± 10 hours
8–9	**Module 9** Flow through orifices	9.1 Small orifices	Exercise 9.1 Summative assessment	± 10 hours
9–10	**Module 10** Conservation of fluid momentum	10.1 Force exerted by a jet of fluid 10.2 Force exerted on reducers and bends	Exercise 10.1 Exercise 10.2 Exercise 10.3 Summative assessment	± 10 hours

Week	Topic	Content	Exercises	Hours
10	**Module 11** Reciprocating pumps	11.1 Types of reciprocating pumps and heads 11.2 Types of pressure heads 11.3 Basic calculations of reciprocating pumps	Exercise 11.1 Summative assessment	± 10 hours
Total				**±100 hours**

9. Lesson plan template

Subject and level	N5 Fluid Mechanics
Prescribed textbook (Title and author)	*N5 Fluid Mechanics* by Peet du Toit

<table>
<tr><th colspan="7">WEEK 1</th></tr>
<tr><th>Content/outcomes to be covered this week</th><th>List of examples to be done in class by the lecturer to explain the outcome/concept</th><th colspan="2">Facilitation method (Please tick)</th><th colspan="2">Teaching resources/aids (Please tick)</th><th>Student activity (exercise in textbook/additional supporting tasks) to be done this week</th></tr>
<tr><td rowspan="8"></td><td rowspan="8"></td><td>Lecture</td><td></td><td>White board/OHP</td><td></td><td rowspan="8"></td></tr>
<tr><td>Group work</td><td></td><td>Models</td><td></td></tr>
<tr><td>Demonstration</td><td></td><td>Handouts</td><td></td></tr>
<tr><td>Simulation</td><td></td><td>Multimedia</td><td></td></tr>
<tr><td colspan="4">Introduction to lessons</td></tr>
<tr><td colspan="4"></td></tr>
<tr><td colspan="4">Recapping/reinforcement</td></tr>
<tr><td colspan="4"></td></tr>
</table>

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WEEK 2

Content/outcomes to be covered this week	List of examples to be done in class by the lecturer to explain the outcome/concept	Facilitation method (Please tick)		Teaching resources/aids (Please tick)		Student activity (exercise in textbook/additional supporting tasks) to be done this week
		Lecture		White board/OHP		
		Group work		Models		
		Demonstration		Handouts		
		Simulation		Multimedia		
		Introduction to lessons				
		Recapping/reinforcement				

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WEEK 3

Content/outcomes to be covered this week	**List of examples to be done in class by the lecturer to explain the outcome/ concept**	**Facilitation method (Please tick)**		**Teaching resources/ aids (Please tick)**		**Student activity (exercise in textbook/additional supporting tasks) to be done this week**
		Lecture		White board/OHP		
		Group work		Models		
		Demonstration		Handouts		
		Simulation		Multimedia		
		Introduction to lessons				
		Recapping/reinforcement				

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WEEK 4

Content/outcomes to be covered this week	**List of examples to be done in class by the lecturer to explain the outcome/concept**	**Facilitation method (Please tick)**		**Teaching resources/aids (Please tick)**		**Student activity (exercise in textbook/additional supporting tasks) to be done this week**
		Lecture		White board/OHP		
		Group work		Models		
		Demonstration		Handouts		
		Simulation		Multimedia		
		Introduction to lessons				
		Recapping/reinforcement				

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WEEK 5

Content/outcomes to be covered this week	**List of examples to be done in class by the lecturer to explain the outcome/ concept**	**Facilitation method (Please tick)**		**Teaching resources/ aids (Please tick)**		**Student activity (exercise in textbook/additional supporting tasks) to be done this week**
		Lecture		White board/OHP		
		Group work		Models		
		Demonstration		Handouts		
		Simulation		Multimedia		
		Introduction to lessons				
		Recapping/reinforcement				

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WEEK 6

Content/outcomes to be covered this week	**List of examples to be done in class by the lecturer to explain the outcome/ concept**	**Facilitation method (Please tick)**		**Teaching resources/ aids (Please tick)**		**Student activity (exercise in textbook/additional supporting tasks) to be done this week**
		Lecture		White board/OHP		
		Group work		Models		
		Demonstration		Handouts		
		Simulation		Multimedia		
		Introduction to lessons				
		Recapping/reinforcement				

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WEEK 7

Content/outcomes to be covered this week	List of examples to be done in class by the lecturer to explain the outcome/ concept	Facilitation method (Please tick)		Teaching resources/ aids (Please tick)		Student activity (exercise in textbook/additional supporting tasks) to be done this week
		Lecture		White board/OHP		
		Group work		Models		
		Demonstration		Handouts		
		Simulation		Multimedia		
		Introduction to lessons				
		Recapping/reinforcement				

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WEEK 8

Content/outcomes to be covered this week	**List of examples to be done in class by the lecturer to explain the outcome/ concept**	**Facilitation method (Please tick)**		**Teaching resources/ aids (Please tick)**		**Student activity (exercise in textbook/additional supporting tasks) to be done this week**
		Lecture		White board/OHP		
		Group work		Models		
		Demonstration		Handouts		
		Simulation		Multimedia		
		Introduction to lessons				
		Recapping/reinforcement				

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WEEK 9

Content/outcomes to be covered this week	List of examples to be done in class by the lecturer to explain the outcome/ concept	Facilitation method (Please tick)		Teaching resources/ aids (Please tick)		Student activity (exercise in textbook/additional supporting tasks) to be done this week
		Lecture		White board/OHP		
		Group work		Models		
		Demonstration		Handouts		
		Simulation		Multimedia		
		Introduction to lessons				
		Recapping/reinforcement				

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WEEK 10

Content/outcomes to be covered this week	List of examples to be done in class by the lecturer to explain the outcome/concept	Facilitation method (Please tick)		Teaching resources/aids (Please tick)		Student activity (exercise in textbook/additional supporting tasks) to be done this week
		Lecture		White board/OHP		
		Group work		Models		
		Demonstration		Handouts		
		Simulation		Multimedia		
		Introduction to lessons				
		Recapping/reinforcement				

MODULE

1 *Properties of fluids*

By the end of this module, students should be able to:

- define fluid; and
- describe and calculate physical properties of fluid in relation to the following:
  - density
  - relative density
  - specific weight
  - viscosity
  - dynamic viscosity
  - kinematic viscosity and surface tension.

Introduction

In this module, which kickstarts the section on hydrostatics, students will become familiar with the properties of fluids. They will learn how to describe and perform calculations for the physical properties of fluids, including density, specific weight and different viscosities.

Exercise 1.1

SB page 6

1.1 $\rho_m = \rho_w \times \rho_r = 1\,000 \times 13{,}6 = 13\,600 \text{ kg/ m}^3$

1.2 $V = \frac{m}{\rho} = \frac{68}{13{,}6 \times 10^3} = 5 \times 10^{-3} \text{ m}^3 = 5\ \ell$

1.3 $\gamma = \rho g = 13{,}6 \times 10^3 \times 9{,}81 = 133{,}416 \text{ kN/ m}^3$

1.4 $m = \rho \times V = 10^3 \times 5 \times 10^{-3} = 5 \text{ kg}$

Exercise 1.2
SB page 7

2.1 $\varepsilon = \frac{\Delta p}{K} = \frac{(10-1)\times 10^6}{1,6\times 10^9} = 5,625\times 10^{-3}$

2.2 $\Delta V = \varepsilon \times V = 5,625\times 10^{-3}\times 5\times 10^{-3} = 5\times 10^{-3} = 28,125\times 10^{-6}\ m^3$

$V_f = V_o - \Delta V = 5\times 10^{-3} - 28,125\times 10^{-6} = 4,972\times 10^{-3}\ m^3 = 4,972\ \ell$

Exercise 1.3
SB page 9

3.1 $\mu = \rho\nu = 850\times 43\times 10^{-6} = 36,55\times 10^{-3}$ Pa.s

3.2 $F = \frac{\mu A v}{t} = \frac{36,55\times 10^{-3}\times 0,5\times 0,3\times 3}{1\times 10^{-3}} = 16,4475$ N

3.3 $\tau = \frac{F}{A} = \frac{16,4475}{0,5\times 0,3} = 109,65$ Pa

Exercise 1.4
SB page 11

4.1 $L = 2\times 0,02 + 2\times 0,06 = 0,16$ m

$F = \sigma L = 72\times 10^{-3}\times 0,16 = 11,52\times 10^{-3}$ N

$m = \frac{F}{g} = \frac{11,52\times 10^{-3}}{9,81} = 1,174\times 10^{-3}$ kg $= 1,174$ g

4.2.1 $L = \frac{F}{\sigma} = \frac{5,34\times 10^{-3}}{425\times 10^{-3}} = 12,565$ mm

$r = \frac{L}{2\pi} = \frac{12,565}{2\pi} = 2$ mm

4.2.2 $p = \frac{2\sigma}{r} = \frac{2\times 425\times 10^{-3}}{2\times 10^{-3}} = 425$ Pa

Summative assessment: Module 1
SB page 14

1.

A	B
1) Density	f) The mass per unit volume ✓
2) Relative density	c) The ratio between the density of the fluid compared to the density of water at 4 °C ✓
3) Specific weight	a) The weight per unit volume ✓
4) Bulk modulus	g) The ratio between the change in pressure and the volumetric strain ✓
5) Dynamic viscosity	e) The measurement of the fluid's resistance to flow ✓
6) Kinematic viscosity	d) The ratio between the dynamic viscosity and the density of the fluid ✓
7) Surface tension	b) The tension of the surface film of a liquid caused by the attraction of the particles ✓

(7)

2. 2.1 $\rho = \frac{\gamma}{g} = \frac{14\,715}{9{,}81}$ ✓ $= 1\,500\ kg/m^3$ ✓ (2)

2.2 $V = \frac{m}{\rho} = \frac{135}{1\,500}$ ✓ $= 0{,}09\ m^3 = 90\ \ell$ ✓ (3)

2.3 $\rho_r = \frac{\rho_s}{\rho_w} = \frac{1\,500}{1\,000}$ ✓ $= 1{,}5$ ✓ (2)

3. 3.1 $\Delta p = \varepsilon K = 5 \times 10^{-3} \times 1{,}66 \times 10^9 = 8\ MPa$ ✓

$p_f = p_o + \Delta p = 1 + 8 = 9\ MPa$ ✓ (3)

3.2 $\Delta V = \varepsilon \times V = 5 \times 10^{-3} \times 5 \times 10^{-3} = 25 \times 10^{-6}\ m^3$ ✓

$V_f = V_o - \Delta V = 5 \times 10^{-3} - 25 \times 10^{-6} = 4{,}975 \times 10^{-3}\ m^3$ ✓

$= 4{,}975\ \ell$ ✓ (3)

4. 4.1 $\mu = \rho\nu = 900 \times 43 \times 10^{-6}$ ✓ $= 38{,}7 \times 10^{-3}\ Pa.s$ ✓ (2)

4.2 $v = \frac{Ft}{\mu A} = \frac{20 \times 2 \times 10^{-3}}{38{,}7 \times 10^{-3} \times 120 \times 10^{-3}}$ ✓ $= 8{,}613\ m/s$ ✓ (2)

4.3 $P = Fv = 20 \times 8{,}613$ ✓ $= 172{,}265\ W$ ✓ (2)

5. $L = \frac{F}{\sigma} = \frac{4 \times 10^{-3} \times 9{,}81}{98{,}1 \times 10^{-3}}$ ✓ $= 0{,}4\ m$ ✓

$D = \frac{L}{\pi} = \frac{0{,}4}{\pi} = 127{,}324\ mm$ ✓ (3)

6. 6.1 $L = \frac{F}{\sigma} = \frac{1 \times 10^{-3} \times 9{,}81}{6 \times 75 \times 10^{-3}} = 21{,}8\ mm$ ✓

$r = \frac{L}{\pi} = \frac{21{,}8}{\pi} = 6{,}939\ mm$ ✓ (2)

6.2 $p = \frac{2\sigma}{r} = \frac{2 \times 75 \times 10^{-3}}{6{,}939 \times 10^{-3}} = 21{,}616\ Pa$ ✓ (1)

7. 7.1 $p = \frac{4\sigma}{r} = \frac{4 \times 72 \times 10^{-3}}{7{,}5 \times 10^{-3}} = 38{,}4\ Pa$ ✓ (1)

7.2 $F = 2\sigma L = 2\sigma \times 2\pi r = 2 \times 72 \times 10^{-3} \times 2\pi \times 7{,}5 \times 10^{-3}$ ✓

$= 6{,}786 \times 10^{-3}\ N$ ✓ (2)

Total: 35 marks

2 *Pressure systems*

By the end of this module, students should be able to:

- describe the application of pressure in fluids with the aid of diagram;
- identify the type of pressure in a system, including:
  - atmospheric pressure
  - gauge pressure
  - vacuum pressure
  - vapour pressure;
- perform calculations relating to the pressure in a system; and
- draw and perform calculations for various pressure measurement instruments, including:
  - barometers
  - piezometers
  - manometers
  - bourdon gauges
  - pitot tubes.

Introduction

Pressure systems refer to regions within a fluid where the pressure varies. Pressure is defined as the force per unit area and is a crucial concept in understanding the behaviour of fluids. Fluids can include liquids and gases. The study of pressure systems helps explain how fluids move and interact within different environments.

Exercise 2.1

SB page 23

1. $p = \rho gh = 900 \times 9{,}81 \times 10 = 88{,}29 \text{ kPa}$
2. $h = \frac{p}{\rho g} = \frac{150 \times 10^3}{1\,000 \times 9{,}81} = 15{,}291 \text{ m}$
3. $p = \rho gh = 13\,600 \times 9{,}81 \times 0{,}7 = 93{,}391 \text{ kPa}$
4. $p_g = p_{abs} - p_a = 80 \times 10^3 - 90 \times 10^3 = -10 \text{ kPa}$

Exercise 2.2 **SB page 28**

1. 1.1 $p = \rho gh = 1\,000 \times 9{,}81 \times 0{,}18 = 1{,}766$ kPa

   1.2 $p_{abs} = p_a + p_g = 100 \times 10^3 + 1{,}766 \times 10^3 = 101{,}766$ kPa

2. $p_A + \rho_1 gh_1 = p_B + \rho_2 gh_2 p_A + 1\,000 \times 9{,}81 \times 0{,}2 = 0 + 13\,600 \times 9{,}81 \times 0{,}5$

   $p_A = 64{,}746$ kPa

3. $p_A + \rho_1 gh_1 = p_B + \rho_2 gh_2 + \rho_3 gh_3 p_A + 800 \times 9{,}81 \times 0{,}5$

   $= p_B + 1\,000 \times 9{,}81 \times 0{,}2 + 13\,600 \times 9{,}81 \times 0{,}25 p_A - p_B = 31{,}392$ kPa

Summative assessment: Module 2 **SB page 31**

1.

A	B
1) Static pressure	e) The pressure exerted by a fluid at rest ✓
2) Atmospheric pressure	a) The pressure exerted on a surface due to the weight of air above it ✓
3) Gauge pressure	d) The difference between the pressure in a fluid and the atmosphere ✓
4) Absolute pressure	c) The pressure being measured from absolute zero ✓
5) Vapour pressure	b) The pressure at which a liquid will start to boil ✓

(5)

2. Pascal's law states that the pressure stays the same in all directions on the same level in a fluid. ✓ (1)

3. A – Liquid under pressure ✓
   B – Fixed end ✓
   C – Oval shaped Bourdon tube ✓
   D – Circular scale ✓
   E – Pointer ✓
   F – Closed free end ✓
   G – Calibration link ✓
   H – Geared sector ✓
   J – Pinion ✓ (9)

4. $p = \rho gh = 800 \times 9{,}81 \times 12 = 94{,}176$ kPa ✓ (1)

5. $h = \frac{p}{\rho g} = \frac{200 \times 10^3}{1\,000 \times 9{,}81} = 20{,}387$ m ✓ (1)

6. $p = \rho gh = 13\,600 \times 9{,}81 \times 0{,}5 = 66{,}708$ kPa ✓ (1)

7. $p_a = p_a - p_g = 80 \times 10^3 - (-20 \times 10^3) = 100$ kPa ✓ (1)

8. $p_A + \rho_1 g h_1 + \rho_2 g h_2 = p_B$ ✓

   $p_A + 13\,600 \times 9{,}81 \times 0{,}05$ ✓ $+ 1\,000 \times 9{,}81 \times 0{,}03$ ✓ $= 0$ ✓

   $p_A = -6{,}965$ kPa ✓ (5)

9. $p_A - \rho_1 g h_1$ ✓ $= p_B - \rho_2 g h_2 - \rho_3 g h_3$

   $p_A - 0{,}8 \times 9{,}81 \times 0{,}4$ ✓ $= 150 - 0{,}9 \times 9{,}81 \times 0{,}1$ ✓ $- 13{,}6 \times 9{,}81 \times 0{,}2$ ✓

   $p_A = 125{,}573$ kPa ✓ (6)

Total: 30 marks

3 Simple hydraulic systems

By the end of this module, students should be able to:

- describe and perform calculations of a simple hydraulics system, including:
  - actuators
  - hydraulics presses in terms of the following:
    - power transfer concepts
    - work done
    - efficiency
    - mechanical advantage;
- understand distortion of hydraulic components due to fluid pressure, effective bulk modulus and free play; and
- perform calculations on rotating shafts or pistons.

Introduction

A simple hydraulic system refers to a basic mechanism that uses the principles of fluid dynamics to transmit force or energy. Hydraulic systems use a fluid, typically oil or water, to transmit power from one point to another. These systems are widely used in various applications, ranging from heavy machinery and industrial equipment to automotive systems and small tools. Before the operation of any hydraulic system can be discussed, a good understanding of the term "pressure" is required. Pressure is the force exerted on a body per unit area.

Exercise 3.1

SB page 38

1. 1.1 $A = \frac{\pi d^2}{4} = \frac{\pi \times 0{,}15^2}{4} = 17{,}671 \times 10^{-3}\ \text{m}^2$

$p = \frac{F}{A} = \frac{4 \times 10^3}{17{,}671 \times 10^{-3}} = 226{,}354\ \text{kPa}$

1.2 $W_d = F \times \ell = 4 \times 10^3 \times 0{,}3 = 1{,}2\ \text{kJ}$

1.3 $v = \frac{P}{F} = \frac{400}{4\ 000} = 0{,}1\ \text{m/s}$

$t = \frac{\ell}{v} = \frac{0{,}3}{0{,}1} = 3\ \text{sec}$

2. $A_1 = \frac{\pi d^2}{4} = \frac{\pi \times 0{,}3^2}{4} = 70{,}686 \times 10^{-3}\ m^2$

$A_2 = \frac{\pi(D^2 - d^2)}{4} = \frac{\pi \times (0{,}3^2 - 0{,}05^2)}{4} = 68{,}722 \times 10^{-3}\ m^2$

$\Delta A = A_1 - A_2 = 1{,}963 \times 10^{-3}\ m^2$

$\Delta F = p \times \Delta A = 1 \times 10^6 \times 1{,}963 \times 10^{-3} = 1{,}963\ kN$

3. 3.1 $A_2 = \frac{\pi D^2}{4} = \frac{\pi \times 0{,}25^2}{4} = 49{,}087 \times 10^{-3}\ m^2$

$p = \frac{F_2}{A_2} = \frac{3\ 000}{49{,}087 \times 10^{-3}} = 61{,}115\ kPa$

3.2 $A_1 = \frac{\pi d^2}{4} = \frac{\pi \times 0{,}05^2}{4} = 1{,}963 \times 10^{-3}\ m^2$

$F_1 = p \times A_1 = 61{,}115 \times 10^3 \times 1{,}963 \times 10^{-3} = 120\ N$

3.3 $MA = \frac{F_2}{F_1} = \frac{3\ 000}{120} = 25$ or $MA = \frac{A_2}{A_1} = \frac{49{,}087 \times 10^{-3}}{1{,}963 \times 10^{-3}} = 25$

3.4 $V_1 = A_1 \times \ell_1 = 7{,}854 \times 10^{-3} \times 0{,}2 = 1{,}571 \times 10^{-3}\ m^3$

$\ell_1 = \frac{A_2 \ell_2}{A_1} = \frac{49{,}087 \times 10^{-3} \times 5 \times 10^{-3}}{1{,}963 \times 10^{-3}} = 125\ mm$

Exercise 3.2

SB page 42

1. $A_2 = \frac{\pi D^2}{4} = \frac{\pi \times 0{,}12^2}{4} = 11{,}31 \times 10^{-3}\ m^2$

$V_2 = A_2 \ell_2 = 11{,}31 \times 10^{-3} \times 88 \times 10^{-3} = 995{,}257 \times 10^{-6}\ m^3$

$V_1 = \frac{V_2}{n \times C_d} = \frac{995{,}257 \times 10^{-6}}{52 \times 0{,}955} = 20{,}041 \times 10^{-6}\ m^3$

2. $A_1 = \frac{V_1}{\ell_1} = \frac{20{,}041 \times 10^{-6}}{40 \times 10^{-3}} = 501{,}035 \times 10^{-6}\ m^2$

$d = \sqrt{\frac{4 A_1}{\pi}} = \sqrt{\frac{4 \times 501{,}035 \times 10^{-6}}{\pi}} = 25{,}257\ mm$

3. $F_2 = mg = 1{,}4 \times 10^3 \times 9{,}81 = 13{,}734\ kN$

$p = \frac{F_2}{A_2} = \frac{13{,}734 \times 10^3}{11{,}31 \times 10^{-3}} = 1{,}214\ MPa$

$F_1 = p \times A_1 = 1{,}214 \times 10^6 \times 501{,}035 \times 10^{-6} = 608{,}433\ N$

$E = \frac{F_1}{\eta \times MA} = \frac{608{,}433}{0{,}93 \times 35} = 18{,}692\ N$

Exercise 3.3 **SB page 44**

1. $A_1 = \frac{\pi D^2}{4} = \frac{\pi \times 0{,}4^2}{4} = 125{,}664 \times 10^{-3}\ m^2$

$A_2 = \frac{\pi d^2}{4} = \frac{\pi \times 0{,}05^2}{4} = 1{,}963 \times 10^{-3}\ m^2$

$F_2 = p_2 A_2 = 128 \times 10^6 \times 1{,}963 \times 10^{-3} = 251{,}327\ kN$

$p_1 = \frac{F_1}{A_1} = \frac{251{,}327 \times 10^3}{125{,}664 \times 10^{-3}} = 2\ MPa$

2. $V_1 = A_1 \ell_1 = 125{,}664 \times 10^{-3} \times 0{,}3 = 37{,}699 \times 10^{-3}\ m^3 = 37{,}699\ \ell$

3. $V_2 = A_2 \times \ell_2 = 1{,}963 \times 10^{-3} \times 0{,}3 = 589{,}049 \times 10^{-6}\ m^3 = 0{,}589\ \ell$

4. $W_d = F \times \ell = 251{,}327 \times 10^3 \times 0{,}3 = 75{,}398\ kJ$

Exercise 3.4 **SB page 47**

1.1 $K_c = \frac{E}{2{,}5} = \frac{180 \times 10^9}{2{,}5} = 72\ GPa$

$K_e = \left(\frac{1}{K_\ell} + \frac{1}{K_c}\right)^{-1} = \left(\frac{1}{1{,}5 \times 10^9} + \frac{1}{72 \times 10^9}\right)^{-1} = 1{,}469\ GPa$

1.2 $K_g = \gamma p = 1{,}4 \times 800 \times 10^3 = 1{,}12\ MPa$

$$K_e = \left(\frac{1}{K_\ell} + \frac{1}{K_c} + \frac{1}{K_g} \times \frac{V_g}{V_t}\right)^{-1}$$

$$= \left(\frac{1}{1{,}5 \times 10^9} + \frac{1}{72 \times 10^9} + \frac{1}{1{,}12 \times 10^6} \times \frac{3}{100}\right)^{-1}$$

$K_e = 36{,}408\ MPa$

2.1 $K_c = \frac{E}{2{,}5} = \frac{200 \times 10^9}{2{,}5} = 80\ GPa$

$K_e = \left(\frac{1}{K_\ell} + \frac{1}{K_c}\right)^{-1} = \left(\frac{1}{2{,}3 \times 10^9} + \frac{1}{80 \times 10^9}\right)^{-1} = 2{,}236\ GPa$

$\Delta V = \frac{p \times V}{K_e} = \frac{1 \times 10^6 \times 0{,}2}{2{,}236 \times 10^9} = 89{,}457 \times 10^{-6}\ m^3$

2.2 $K_g = \gamma p = 1{,}4 \times 1 \times 10^6 = 1{,}4\ MPa$

$$K_e = \left(\frac{1}{K_\ell} + \frac{1}{K_c} + \frac{1}{K_g} \times \frac{V_g}{V_t}\right)^{-1}$$

$$= \left(\frac{1}{2{,}3 \times 10^9} + \frac{1}{80 \times 10^9} + \frac{1}{1{,}4 \times 10^6} \times \frac{2}{100}\right)^{-1}$$

$K_e = 45{,}713\ MPa$

$\Delta V = \frac{p \times V}{K_e} = \frac{1 \times 10^6 \times 0{,}2}{45{,}713 \times 10^6} = 6{,}125 \times 10^{-3}\ m^3$

Exercise 3.5 SB page 50

1.1 $F = \frac{T}{r} = \frac{50}{0,04} = 1,25 \text{ kN}$

1.2 $A = \pi d\ell = \pi \times 0,08 \times 0,07 = 17,593 \times 10^{-3} \text{ m}^2$

$$v = \frac{Ft}{\mu A} = \frac{1,2 \times 10^3 \times 0,2 \times 10^{-3} \times 1,571}{0,4 \times 17,593 \times 10^{-3}} = 35,526 \text{ m/s}$$

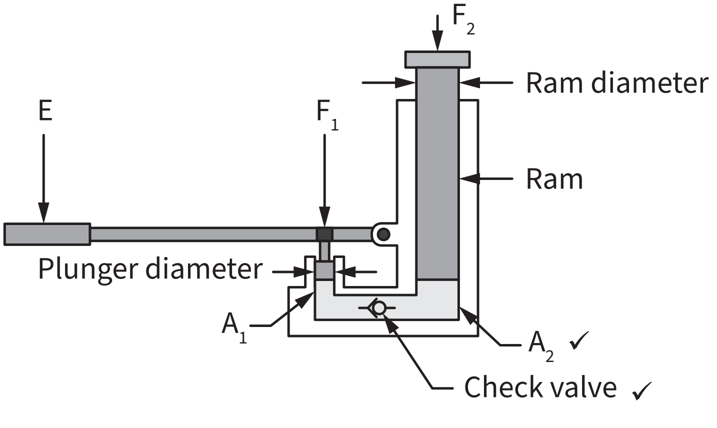
1.3 $N = \frac{v}{\pi d} = \frac{35,526}{\pi \times 0,08} = 141,352 \text{ revs/s}$

Summative assessment: Module 3 SB page 52

1. Pressure is the force exerted ✓ on a body per unit area ✓ (2)
2. Pascal's law states that the pressure stays the same in all directions ✓ on the same level in a fluid. ✓ (2)
3. In a single-acting cylinder, the pressure is supplied to one end ✓of the cylinder only. ✓

   In a double-acting cylinder, the pressure could be applied to either side of the piston ✓ resulting in different forces to be applied ✓ (4)

4.



(6)

5.1 $A_2 = \frac{\pi D^2}{4} = \frac{\pi \times 0,1^2}{4} = 7,853 \times 10^{-3} \text{ m}^2$ ✓ ✓ ✓

$F_2 = p \times A_2 = 2 \times 10^6 \times 7,853 \times 10^{-3} = 15,708 \text{ kN}$ ✓

$m = \frac{F_2}{g} = \frac{15,708 \times 10^3}{9,81} = 1,601 \text{ Mg}$ ✓ ✓ (3)

5.2 $V_p = \frac{A_1 \ell_1}{n \times C_d} = \frac{7,854 \times 10^{-6} \times 0,1}{30 \times 0,95}$ ✓ $= 27,558 \times 10^{-6} \text{ m}^3$ ✓ ✓ (2)

5.3 $A_1 = \frac{V_p}{\ell_1} = \frac{27{,}558 \times 10^{-6}}{50 \times 10^{-3}} = 551{,}157 \times 10^{-6}\ \text{m}^2$ ✓

$d = \sqrt{\frac{4A_1}{\pi}} = \sqrt{\frac{4 \times 551{,}157 \times 10^{-6}}{\pi}} = 26{,}491\ \text{mm}$ ✓ (2)

5.4 $F_1 = p \times A_1 = 2 \times 10^6 \times 551{,}157 \times 10^{-6} = 1{,}102\ \text{kN}$ ✓ (1)

5.5 $\eta = \frac{F_1}{E \times MA} = \frac{1{,}102 \times 10^3}{50 \times 25} = 0{,}882 = 88{,}185\%$ ✓ (1)

6.1 $A_1 = \frac{\pi D^2}{4} = \frac{\pi \times 0{,}4^2}{4} = 125{,}664 \times 10^{-3}\ \text{m}^2$ ✓

$F = p_1 \times A_1 = 2 \times 10^6 \times 125{,}664 \times 10^{-3} = 251{,}327\ \text{kN}$ ✓

$d = \sqrt{\frac{4A_1}{\pi}} = \sqrt{\frac{4 \times 1{,}963 \times 10^{-3}}{\pi}} = 50\ \text{mm}$ ✓ (3)

6.2 $A_2 = \frac{F}{p_2} = \frac{251{,}327 \times 10^3}{128 \times 10^6} = 1{,}963 \times 10^{-3}\ \text{m}^2$ ✓

$\ell_2 = \frac{V_2}{A_2} = \frac{1 \times 10^{-3}}{1{,}963 \times 10^{-3}} = 509{,}296\ \text{mm}$ ✓ (2)

6.3 $V_1 = A_1 \ell_1 = 125{,}664 \times 10^{-3} \times 509{,}296 \times 10^{-3} = 64 \times 10^{-3}\ \text{m}^3$ ✓ $= 64\ \ell$ ✓ (2)

6.4 $W_d = F \times \ell = 251{,}327 \times 10^3 \times 509{,}296 \times 10^{-3} = 128\ \text{kJ}$ ✓ (1)

7.1 $K_c = \frac{E}{2{,}5} = \frac{190 \times 10^9}{2{,}5} = 76\ \text{GPa}$ ✓

$K_e = \left(\frac{1}{K_\ell} + \frac{1}{K_c}\right)^{-1} = \left(\frac{1}{1{,}9 \times 10^9} + \frac{1}{76 \times 10^9}\right)^{-1}$ ✓ $= 1{,}854\ \text{GPa}$ ✓ (3)

7.2 $K_g = \gamma p = 1{,}4 \times 700 \times 10^3 = 980\ \text{kPa}$ ✓

$$K_e = \left(\frac{1}{K_\ell} + \frac{1}{K_c} + \frac{1}{K_g} \times \frac{V_g}{V_t}\right)^{-1}$$

$$= \left(\frac{1}{1{,}9 \times 10^9} + \frac{1}{76 \times 10^9} + \frac{1}{980 \times 10^3} \times \frac{2}{100}\right)^{-1}$$ ✓

$K_e = 47{,}738\ \text{MPa}$ ✓ (3)

8.1 $K_c = \frac{E}{2,5} = \frac{180 \times 10^9}{2,5} = 72 \text{ GPa}$ ✓

$K_e = \left(\frac{1}{K_\ell} + \frac{1}{K_c}\right)^{-1} = \left(\frac{1}{2,1 \times 10^9} + \frac{1}{72 \times 10^9}\right)^{-1} = 2,04 \text{ GPa}$ ✓

$\Delta V = \frac{p \times V}{K_e} = \frac{3 \times 10^6 \times 0,4}{2,04 \times 10^9} = 588,095 \times 10^{-6} \text{ m}^3$ ✓

$\Delta\ell = \frac{\Delta V}{\ell} = \frac{588,095 \times 10^{-6}}{0,8} = 0,735 \text{ mm}$ ✓ (4)

8.2 $K_g = \gamma p = 1,4 \times 3 \times 10^6 = 4,2 \text{ MPa}$ ✓

$K_e = \left(\frac{1}{K_\ell} + \frac{1}{K_c} + \frac{1}{K_g} \times \frac{V_g}{V_t}\right)^{-1} = \left(\frac{1}{2,1 \times 10^9} + \frac{1}{72 \times 10^9} + \frac{1}{4,2 \times 10^6} \times \frac{2}{100}\right)^{-1}$

$K_e = 190,404 \text{ MPa}$ ✓

$\Delta V = \frac{p \times V}{K_e} = \frac{3 \times 10^6 \times 0,4}{190,404 \times 10^6} = 6,302 \times 10^{-3} \text{ m}^3$ ✓

$\Delta\ell = \frac{\Delta V}{\ell} = \frac{6,302 \times 10^{-3}}{0,8} = 7,878 \text{ mm}$ ✓ (4)

9.1 $v = \pi dN = \pi \times 0,09 \times 5 = 1,414 \text{ m/s}$ ✓
$A = \pi d\ell = \pi \times 0,09 \times 0,1 = 28,274 \times 10^{-3} \text{ m}^2$ ✓

$t = \frac{D - d}{2} = \frac{90,2 - 90}{2} = 0,1 \text{ mm}$ ✓

$F = \frac{\mu A v}{t} = \frac{0,5 \times 28,274 \times 10^{-3} \times 1,414}{0,1 \times 10^{-3}} = 199,859 \text{ N}$ ✓ (4)

9.2 $T = F \times r = 199,859 \times 0,045 = 8,994 \text{ Nm}$ ✓ (1)

9.3 $P = 2\pi NT = 2\pi \times 5 \times 8,994 = 282,545 \text{ W}$ ✓ (1)

10.1 $F = \frac{T}{r} = \frac{30}{0,05} = 600 \text{ N}$ ✓ (1)

10.2 $A = \frac{Ft}{\mu v} = \frac{600 \times 0,2 \times 10^{-3}}{0,5 \times 10} = 24 \times 10^{-3} \text{ m}^2$ ✓

$\ell = \frac{A}{\pi d} = \frac{24 \times 10^{-3}}{\pi \times 0,1} = 76,394 \text{ mm}$ ✓ (2)

10.3 $P = Fv = 600 \times 10 = 6 \text{ kW}$ ✓ (1)

Total: 55 marks

MODULE

4 *Hydrostatic forces on submerged areas*

By the end of this module, students should be able to:

- perform calculations related to:
  - sluice-gates with upper surface on the same level with fluid level;
  - HSF and the centre of pressure referenced from the top or bottom; and
  - the total force and the centre of pressure referenced from the top or the bottom.

Introduction

In Module 2, students were introduced to the pressure distribution diagram and formula for static pressure exerted by a liquid. This module deals with the hydrostatic force caused by the static pressure of a liquid.

Exercise 4.1

SB page 58

1.1 $p = \rho g h = 1\,000 \times 9{,}81 \times 5 = 49{,}05 \text{ kPa}$

$A = 3 \times 3 = 9 \text{ m}^2$

$F = p \times A = 49{,}05 \times 10^3 \times 9 = 441{,}45 \text{ kN}$

1.2 $p = \rho g h_c = 1\,000 \times 9{,}81 \times 2{,}5 = 24{,}525 \text{ kPa}$

$A = 3 \times 5 = 15 \text{ m}^2$

$F = p \times A = 224{,}525 \times 10^3 \times 15 = 367{,}875 \text{ kN}$

1.3 $h_p = \frac{2h}{3} = \frac{2 \times 5}{3} = 3{,}333 \text{ m (from top)}$

2.1 $p = \rho g h = 1\,000 \times 9{,}81 \times 4 = 39{,}24 \text{ kPa}$

$A = \frac{\pi d^2}{4} = \frac{\pi \times 3^2}{4} = 7{,}069 \text{ m}^2$

$F = p \times A = 39{,}24 \times 10^3 \times 7{,}069 = 277{,}371 \text{ kN}$

2.2 $p = \rho g h_c = 1\,000 \times 9{,}81 \times 2 = 19{,}62$ kPa

$A = \pi dh = \pi \times 3 \times 4 = 37{,}699\ m^2$

$F = p \times A = 19{,}62 \times 10^3 \times 37{,}699 = 739{,}657$ kN

2.3 $h_p = \frac{1h}{3} = \frac{1 \times 4}{3} = 1{,}333$ m (from bottom)

Exercise 4.2

SB page 62

1.1 Consider the air:

$A_a = 4 \times 3 = 12\ m^2$ (area which the air acts on)

$F_a = p_a \times A_a = 40 \times 10^3 \times 12 = 480$ kN

Consider the water:

$p_w = \rho g h_c = 1\,000 \times 9{,}81 \times 2 = 19{,}62$ kPa

$A_w = 2 \times 3 = 6\ m^2$ (area which the oil acts on)

$F_w = p_w \times A_w = 19{,}62 \times 10^3 \times 6 = 117{,}72$ kN

$F_T = F_a + F_w = 480 \times 10^3 + 117{,}72 \times 10^3 = 597{,}72$ kN

1.2 $h_{pa} = \frac{H}{2} = \frac{4}{2} = 2$ m (from bottom)

$h_{pw} = \frac{h}{3} = \frac{2}{3} = 0{,}667$ m (from bottom)

Take moments about the bottom:

$F_T \times \bar{h} = F_a \times h_{pa} + F_w \times h_{po}$ $597{,}72 \times \bar{h} = 480 \times 2 + 117{,}72 \times 0{,}667$

$\bar{h} = 1{,}737$ m (from bottom)

2.1 Consider the water:

$p_w = \rho g h_c = 1\,000 \times 9{,}81 \times 1{,}5 = 14{,}715$ kPa

$A_w = \pi d h_w = \pi \times 3 \times 3 = 28{,}274\ m^2$ (area which the water acts on)

$F_w = p_w \times A_w = 14{,}715 \times 10^3 \times 28{,}274 = 416{,}057$ kN

Consider the oil:

$p_o = \rho g h_c = 900 \times 9{,}81 \times 0{,}9 = 7{,}946$ kPa

$A_o = \pi d h_o = \pi \times 3 \times 1{,}8 = 16{,}965\ m^2$ (area which the oil acts on)

$F_{o1} = p_o \times A_o = 7{,}946 \times 10^3 \times 16{,}965 = 134{,}802$ kN

Consider the effect of the oil pressure on the water surface:

$p = \rho g h_o = 900 \times 9{,}81 \times 1{,}8 = 15{,}892$ kPa

$F_{o2} = p \times A_w = 15{,}892 \times 10^3 \times 28{,}274 = 449{,}341$ kN

$F_T = F_w + F_{o1} + F_{o2} = 416{,}057 \times 10^3 + 134{,}802 \times 10^3 + 449{,}341 \times 10^3$

$= 1$ MN

2.2 $h_{pw} = 1{,}8 + \frac{2h_w}{3} = 1{,}8 + \frac{2 \times 3}{3} = 3{,}8$ m (from top)

$h_{po1} = \frac{2h_o}{3} = \frac{2 \times 1{,}8}{3} = 1{,}2$ m (from top)

$h_{po2} = 1{,}8 + \frac{h_w}{2} = 1{,}8 + \frac{3}{2} = 3{,}3$ m (from top)

Take moments about the top:

$F_T \times \bar{h} = F_w \times h_{pw} + F_{o1} \times h_{po1} + F_{o2} \times h_{po2}$

$1 \times 10^3 \times \bar{h} = 416{,}057 \times 3{,}8 + 134{,}802 \times 1{,}2 + 449{,}341 \times 3{,}3$

$\bar{h} = 3{,}225$ m (from top)

Exercise 4.3

SB page 68

1.1 $h_c = \frac{d}{2} = \frac{2}{2} = 1$ m

$A = bd = 4 \times 2 = 8$ m^2

$F = \rho g h_c A = 1\,000 \times 9{,}81 \times 1 \times 8 = 78{,}48$ kN

1.2 $I_c = \frac{bd^3}{12} = \frac{4 \times 2^3}{12} = 2{,}667$ m^4

$h_p = h_c + \frac{I_c}{Ah_c} = 1 + \frac{2{,}667}{8 \times 1} = 1{,}333$ m

2.1 $h_c = 3 + \frac{d}{2} = 3 + \frac{2}{2} = 4$ m

$A = bd = 4 \times 2 = 8$ m^2

$F = \rho g h_c A = 1\,000 \times 9{,}81 \times 4 \times 8 = 313{,}92$ kN

2.2 $I_c = \frac{bd^3}{12} = \frac{4 \times 2^3}{12} = 2{,}667$ m^4

$h_p = h_c + \frac{I_c}{Ah_c} = 4 + \frac{2{,}667}{8 \times 4} = 4{,}083$ m

3.1 $h_c = 2 + \frac{D}{2} = 2 + \frac{1}{2} = 2{,}5$ m

$A = \frac{\pi D^2}{4} = \frac{\pi \times 1^2}{4} = 0{,}785$ m^2

$F = \rho g h_c A = 850 \times 9{,}81 \times 2{,}5 \times 0{,}785 = 16{,}373$ kN

3.2 $I_c = \frac{\pi D^4}{64} = \frac{\pi \times 1^4}{64} = 0{,}049 \text{ m}^4$

$h_p = h_c + \frac{I_c}{A h_c} = 2{,}5 + \frac{0{,}049}{0{,}785 \times 2{,}5} = 2{,}525 \text{ m}$

4.1 $h_c = 2 + \frac{d}{2} = 2 + \frac{4}{2} = 4 \text{ m}$

$A = bd = 4 \times 4 = 16 \text{ m}^2$

$F = \rho g h_c A = 1\,000 \times 9{,}81 \times 4 \times 16 = 627{,}84 \text{ kN}$

4.2 $I_c = \frac{bd^3}{12} = \frac{4 \times 4^3}{12} = 21{,}333 \text{ m}^4$

$h_p = h_c + \frac{I_c}{A h_c} = 4 + \frac{21{,}333}{16 \times 4} = 4{,}333 \text{ m}$

4.3 Take moments about H:

$W \times x = F_w \times y_p \, 1 \times 10^6 \times x = 627{,}84 \times 10^3 \times 4{,}333 x = 1{,}465 \text{ m}$

5.1 $A = \frac{\pi D^2}{4} = \frac{\pi \times 4^2}{4} = 12{,}566 \text{ m}^2$

$h_c = r \times \sin\theta = 2 \times \sin 45 = 1{,}414 \text{ m}$

$F = \rho g h_c A = 1\,000 \times 9{,}81 \times 1{,}414 \times 12{,}566 = 174{,}339 \text{ kN}$

5.2 $I_c = \frac{\pi D^4}{64} = \frac{\pi \times 4^4}{64} = 12{,}566 \text{ m}^4$

$h_p = h_c + \frac{I_c \sin^2\theta}{A h_c} = 1{,}414 + \frac{12{,}566 \times (\sin 45)^2}{12{,}566 \times 1{,}414} = 1{,}768 \text{ m}$

6.1 $A = bd = 5 \times 4 = 20 \text{ m}^2$

$h_c = 3 + \frac{d}{2} \times \sin\theta = 3 + 2 \times \sin 60 = 4{,}732 \text{ m}$

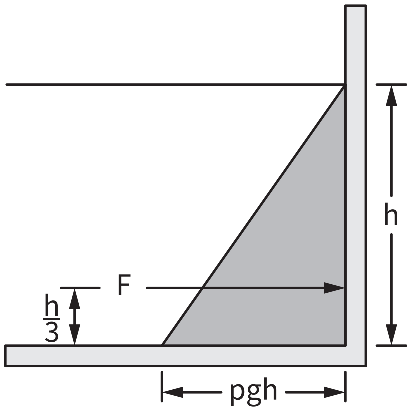
$F = \rho g h_c A = 1\,000 \times 9{,}81 \times 4{,}732 \times 20 = 928{,}428 \text{ kN}$

6.2 $I_c = \frac{bd^3}{12} = \frac{5 \times 4^3}{12} = 26{,}667 \text{ m}^4$

$h_p = h_c + \frac{I_c \sin^2\theta}{A h_c} = 4{,}732 + \frac{26{,}667 \times (\sin 60)^2}{20 \times 4{,}732} = 4{,}943 \text{ m}$

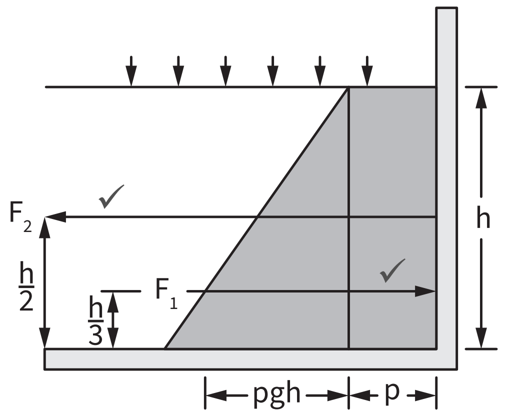
Summative assessment: Module 4 **SB page 71**

1.1



(3)

1.2



(3)

2. The distance to the centroid is measured from the surface to the centroid of the object. ✓ This value is used to calculate the average pressure acting on the object. ✓

The centre of pressure position is the distance from the surface to the point where the hydrostatic force ✓ is concentrated. ✓ (4)

3.1 $p = \rho gh = 1\ 000 \times 9{,}81 \times 3 = 29{,}43$ kPa ✓

$A = 5 \times 4 = 20\ m^2$

$F = p \times A = 29{,}43 \times 10^3 \times 20 = 588{,}6$ kN ✓ (2)

3.2 $p = \rho g h_c = 1\ 000 \times 9{,}81 \times 1{,}5 = 14{,}715$ kPa ✓

$A = 3 \times 5 = 15\ m^2$

$F = p \times A = 14{,}715 \times 10^3 \times 15 = 220{,}725$ kN ✓ (2)

3.3 $h_p = \frac{2h}{3} = \frac{2 \times 3}{3} = 2$ m (from top) ✓ (1)

4.1 $p = \rho gh = 1\,000 \times 9{,}81 \times 4{,}5 = 44{,}145$ kPa ✓

$A = \frac{\pi d^2}{4} = \frac{\pi \times 1{,}5^2}{4} = 1{,}767\ m^2$

$F = p \times A = 44{,}415 \times 10^3 \times 1{,}767 = 78{,}011$ kN ✓ (2)

4.2 $p = \rho g h_c = 1\,000 \times 9{,}81 \times 2{,}25 = 22{,}073$ kPa ✓

$A = \pi dh = \pi \times 1{,}5 \times 4{,}5 = 21{,}206\ m^2$

$F = p \times A = 22{,}073 \times 10^3 \times 21{,}206 = 468{,}064$ kN ✓ (2)

4.3 $h_p = \frac{2h}{3} = \frac{3 \times 4{,}5}{3} = 4{,}5$ m (from top) ✓ (1)

5.1 Consider the air:

$A_a = 4 \times 2 = 8\ m^2$ (area which the air acts on) ✓

$F_a = p_a \times A_a = 20 \times 10^3 \times 8 = 160$ kN ✓

Consider the water:

$p_w = \rho g h_c = 1\,000 \times 9{,}81 \times 0{,}75 = 7{,}358$ kPa ✓

$A_w = 1{,}5 \times 4 = 6\ m^2$ (area which the water acts on)

$F_w = p_w \times A_w = 7{,}358 \times 10^3 \times 6 = 44{,}145$ kN ✓

$F_T = F_a + F_w = 160 \times 10^3 + 44{,}145 \times 10^3 = 204{,}145$ kN ✓ (5)

5.2 $h_{pa} = \frac{H}{2} = \frac{2}{2} = 1$ m (from top)

$h_{pw} = 0{,}5 + \frac{2h}{3} = 0{,}5 + \frac{2 \times 1{,}5}{3} = 1{,}5$ m (from top) ✓

Take moments about the top:

$F_T \times \bar{h} = F_a \times h_{pa} + F_w \times h_{po}$

$204{,}145 \times \bar{h} = 160 \times 1 + 44{,}145 \times 1{,}5$ ✓

$\bar{h} = 1{,}108$ m (from top) ✓ (3)

6.1 Consider the water:

$p_w = \rho g h_c = 1\,000 \times 9{,}81 \times 0{,}5 = 4{,}905$ kPa ✓

$A_w = \pi d h_w = \pi \times 3 \times 1 = 9{,}425\ m^2$ (area which the water acts on)

$F_w = p_w \times A_w = 4{,}905 \times 10^3 \times 9{,}425 = 46{,}229$ kN ✓

Consider the oil:

$p_o = \rho g h_c = 900 \times 9{,}81 \times 0{,}6 = 5{,}297$ kPa ✓

$A_o = \pi d h_o = \pi \times 3 \times 1{,}2 = 11{,}31$ m^2 (area which the oil acts on)

$F_{o1} = p_o \times A_o = 5{,}297 \times 10^3 \times 11{,}31 = 59{,}912$ kN ✓

Consider the effect of the oil pressure on the water surface:

$p = \rho g h_o = 900 \times 9{,}81 \times 1{,}2 = 10{,}595$ kPa ✓

$F_{o2} = p \times A_w = 10{,}595 \times 10^3 \times 9{,}425 = 99{,}854$ kN ✓

$F_T = F_w + F_{o1} + F_{o2} = 46{,}229 + 59{,}912 + 99{,}854 = 205{,}994$ kN ✓ (7)

6.2 $h_{pw} = 1{,}2 + \frac{2h_w}{3} = 1{,}2 + \frac{2 \times 1}{3} = 1{,}867$ m (from top)

$h_{po1} = \frac{2h_o}{3} = \frac{2 \times 1{,}2}{3} = 0{,}8$ m (from top)

$h_{po2} = 1{,}2 + \frac{h_w}{2} = 1{,}2 + \frac{1}{2} = 1{,}7$ m (from top)

Take moments about the top:

$F_T \times \bar{h} = F_w \times h_{pw} + F_{o1} \times h_{po1} + F_{o2} \times h_{po2}$

$205{,}994 \times \bar{h}$ ✓ $= 46{,}229 \times 1{,}867 + 59{,}912 \times 0{,}8 + 99{,}854 \times 1{,}7$ ✓

$\bar{h} = 1{,}476$ m (from top) ✓ (3)

7.1 $h_c = \frac{d}{2} = \frac{3}{2} = 1{,}5$ m ✓

$A = bd = 3 \times 2 = 6$ m^2

$F = \rho g h_c A = 1\,000 \times 9{,}81 \times 1{,}5 \times 6 = 88{,}29$ kN ✓ (2)

7.2 $h_p = \frac{2h}{3} = \frac{2 \times 3}{3} = 2$ m ✓ (1)

8.1 $h_c = 1 + \frac{d}{2} = 1 + \frac{3}{2} = 2{,}5$ m ✓

$A = bd = 3 \times 2 = 6$ m^2 ✓

$F = \rho g h_c A = 1\,000 \times 9{,}81 \times 2{,}5 \times 6 = 147{,}15$ kN ✓ (3)

8.2 $I_c = \frac{bd^3}{12} = \frac{2 \times 3^3}{12} = 1{,}5$ m^4 ✓

$h_p = h_c + \frac{I_c}{A h_c} = 2{,}5 + \frac{1{,}5}{6 \times 2{,}5} = 2{,}6$ m ✓

9.1 $h_c = 2 + \frac{d}{2} = 2 + \frac{1,5}{2} = 2,75 \text{ m}$ ✓

$A = bd = 1,5 \times 3 = 4,5 \text{ m}^2$ ✓

$F = \rho g h_c A = 1\,030 \times 9,81 \times 2,75 \times 4,5 = 125,041 \text{ kN}$ ✓ (3)

9.2 $I_c = \frac{bd^3}{12} = \frac{3 \times 1,5^3}{12} = 0,844 \text{ m}^4$ ✓

$h_p = h_c + \frac{I_c}{Ah_c} = 2,75 + \frac{0,844}{4,5 \times 2,75} = 2,818 \text{ m}$ ✓ (2)

9.3 Take moments about H:

$F_A \times x = F_w \times y_p$

$F_A \times 1,5 = 125,041 \times 10^3 \times 0,818$ ✓

$F_A = 68,204 \text{ kN}$ ✓ (2)

10.1 $A = \frac{\pi D^2}{4} = \frac{\pi \times 2^2}{4} = 3,142 \text{ m}^2$ ✓

$I_c = \frac{\pi D^4}{64} = \frac{\pi \times 2^4}{64} = 0,785 \text{ m}^4$ ✓

$y_p = r + \frac{I_c}{Ah_c} = 1 + \frac{0,785}{3,142 \times h_c}$ ✓

$F = \rho g h_c A = 1\,000 \times 9,81 \times h_c \times 3,142 = 30,819 \times 10^3\ h_c$ ✓

Take moments about H:

$W \times x = F_w \times y_p\, 32 \times 10^3 \times 2 = 30,819 \times 10^3\ h_c \times \left(1 + \frac{0,785}{3,142 \times h_c}\right)$ ✓

$64 \times 10^3 = 30,819 \times 10^3\ h_c + 7,705 \times 10^3$

$h_c = 1,827 \text{ m}$ ✓

$h_1 = h_c - 1 = 1,827 - 1 = 0,827 \text{ m}$ ✓ (7)

10.2 $F_w = 30,819 \times 10^3\ h_c = 30,819 \times 10^3 \times 1,827 = 56,295 \text{ kN}$ ✓ (1)

10.3 $h_p = h_c + \frac{I_c}{Ah_c} = 1,827 + \frac{0,785}{3,142 \times 1,827}$ ✓ $= 1,964 \text{ m}$ ✓ (2)

11.1 $A = bd = 0,4 \times 0,4 = 0,16 \text{ m}^2$ ✓

$h_c = 0,35 + \frac{0,4}{2} = 0,55 \text{ m}$ ✓

$F = \rho g h_c A = 1\,000 \times 9,81 \times 0,55 \times 0,16 = 863,28 \text{ N}$ ✓ (3)

11.2 $I_c = \frac{bd^3}{12} = \frac{0,4 \times 0,4^3}{12} = 2,133 \times 10^{-3} \text{ m}^4$ ✓

$h_{pw} = h_c + \frac{I_c}{Ah_c} = 0,55 + \frac{2,133 \times 10^{-3}}{0,16 \times 0,55} = 0,574 \text{ m}$ ✓ (2)

11.3 $y_{po} = \frac{d}{2} + \frac{I_c}{Ah_c} = 0{,}2 + \frac{2{,}133 \times 10^{-3}}{0{,}16 \times h_c}$ ✓

$F_o = \rho g h_c A = 800 \times 9{,}81 \times h_c \times 0{,}16 = 1{,}256 \times 10^3\ h_c$ ✓

$y_{pw} = h_{pw} - 0{,}35 = 0{,}574 - 0{,}35 = 0{,}224$ m ✓

Take moments about H:

$F_o \times y_{po} = F_w \times y_{pw}\ 1{,}256 \times 10^3\ h_c \times \left(0{,}2 + \frac{2{,}133 \times 10^{-3}}{0{,}16 \times h_c}\right)$

$= 863{,}28 \times 0{,}224$ ✓

$251{,}136\ h_c + 16{,}7424 = 193{,}584$

$h_c = 0{,}704$ m ✓

$h_1 = h_c - 0{,}2 = 0{,}704 - 0{,}2 = 0{,}504$ m ✓ (height above door)

$h_o = h_1 + 0{,}4 + 0{,}25 = 0{,}504 + 0{,}65 = 1{,}154$ m ✓ (7)

12.1 $A_{AB} = bd = 1 \times 0{,}8 = 0{,}8\ m^2$ ✓

$h_c = 1$ m

$F_{AB} = \rho g h_c A = 1\,000 \times 9{,}81 \times 1 \times 0{,}8 = 7{,}848$ kN ✓ (2)

12.2 $A_{BC} = bd = 2 \times 0{,}8 = 1{,}6\ m^2$ ✓

$h_c = 1 + \frac{d}{2} = 1 + \frac{2}{2} = 2$ m ✓

$F_{BC} = \rho g h_c A = 1\,000 \times 9{,}81 \times 2 \times 1{,}6 = 31{,}392$ kN ✓ (3)

12.3 Consider BC:

$I_c = \frac{bd^3}{12} = \frac{0{,}8 \times 2^3}{12} = 0{,}533\ m^4$ ✓

$h_p = h_c + \frac{I_c}{Ah_c} = 2 + \frac{0{,}533}{1{,}6 \times 2} = 2{,}167$ m ✓

$y_{BC} = h_p - 1 = 2{,}167 - 1 = 1{,}167$ m ✓

Take moments about A:

$W \times 1 = F_{AB} \times 0{,}5 + F_{BC} \times 1{,}167$

$W \times 1 = 31{,}392 \times 0{,}5 + 7{,}848 \times 1{,}167$ ✓

$W = 40{,}548$ kN ✓ (5)

13.1 $A = \frac{\pi D^2}{4} = \frac{\pi \times 3^2}{4} = 7{,}068\ m^2$ ✓

$h_c = r \times \sin\theta = 1{,}5 \times \sin 45 = 1{,}061$ m ✓

$F = \rho g h_c A = 1\,000 \times 9{,}81 \times 1{,}061 \times 7{,}068 = 73{,}549$ kN ✓ (3)

13.2 $I_c = \frac{\pi D^4}{64} = \frac{\pi \times 3^4}{64} = 3{,}976\ m^4$ ✓

$h_p = h_c + \frac{I_c \sin^2\theta}{A h_c} = 1{,}061 + \frac{3{,}976 \times (\sin 45)^2}{7{,}068 \times 1{,}061} = 1{,}326\ m$ ✓ (2)

14.1 $A = bd = 5 \times 6 = 30\ m^2$ ✓

$h_c = 4 + \frac{d}{2} \times \sin\theta = 4 + 2{,}5 \times \sin 50 = 5{,}915\ m$ ✓

$F = \rho g h_c A = 1\ 000 \times 9{,}81 \times 5{,}915 \times 30 = 1{,}741\ MN$ ✓ (3)

14.2 $I_c = \frac{b d^3}{12} = \frac{6 \times 5^3}{12} = 62{,}5\ m^4$ ✓

$h_p = h_c + \frac{I_c \sin^2\theta}{A h_c} = 5{,}915 + \frac{62{,}5 \times (\sin 50)^2}{30 \times 5{,}915} = 6{,}122\ m$ ✓ (2)

Total: 95 marks

5 Buoyancy and stability of floating and immersed bodies

By the end of this module, students should be able to:
- explain Archimedes' principle; and
- perform calculations on:
  - buoyancy for spherical- and cubic shaped bodies
  - metacentric height and righting moments on ships.

Introduction

A solid body dropped into a fluid will sink, float, or remain at rest at any point in the fluid, depending on its density relative to the density of the fluid. Note that when the relative density is equal to or greater than one, the floating body becomes completely submerged. It follows from these discussions that a body immersed in a fluid remains at rest at any point in the fluid when its density is equal to the density of the fluid, sinks to the bottom when its density is greater than the density of the fluid, and rises to the surface of the fluid and floats when the density of the body is less than the density of the fluid.

Exercise 5.1

SB page 82

1.1 $V_w = A \times \ell = 6 \times 1{,}5 \times 12 = 108 \text{ m}^3$

$F_B = \rho g V_\ell = 1\,000 \times 9{,}81 \times 108 = 1{,}059 \text{ MN} = W$

$m = \frac{W}{g} = \frac{1{,}059 \times 10^6}{9{,}81} = 108 \times 10^3 \text{ kg or } 108 \text{ t}$

1.2 $V_{sw} = \frac{F_B}{\rho g} = \frac{1{,}059 \times 10^6}{1030 \times 9{,}81} = 104{,}854 \text{ m}^3$

$d = \frac{V_{sw}}{A} = \frac{104{,}854}{6 \times 12} = 1{,}456 \text{ m}$

2.1 $V_s = \frac{m}{\rho} = \frac{1{,}3}{750} = 1{,}733 \times 10^{-3} \text{ m}^3$

$D = \sqrt[3]{\frac{6V_s}{\pi}} = \sqrt[3]{\frac{6 \times 1{,}733 \times 10^{-3}}{\pi}} = 149{,}037 \text{ mm}$

2.2 $V_w = \frac{m}{\rho} = \frac{1,3}{1\,000} = 1,3 \times 10^{-3}\ m^3 = 1,3\ \ell$

2.3 $\%V = \frac{V_W}{V_s} = \frac{1,3 \times 10^{-3}}{1,733 \times 10^{-3}} = 0,75 = 75\%$

3.1 $V_w = \frac{\pi \times D^2 \times \ell}{4} = \frac{\pi \times 0,72^2 \times 1}{4} = 407,15 \times 10^{-3}\ m^3$

$F_B = \rho g V_w = 10^3 \times 9,81 \times 407,15 \times 10^{-3} = 3,994\ kN/m$

3.2 $V_s = \frac{\pi \times (D^2 - d^2) \times \ell}{4} = \frac{\pi \times (0,72^2 - 0,7^2) \times 1}{4} = 22,305 \times 10^{-3}\ m^3$

$W_s = \rho g V_s = 7\,800 \times 9,81 \times 22,305 \times 10^{-3} = 1,707\ kN/m$

$V_o = \frac{\pi \times d^2 \times \ell}{4} = \frac{\pi \times 0,7^2 \times 1}{4} = 384,845 \times 10^{-3}\ m^3$

$W_o = \rho g V_o = 600 \times 9,81 \times 384,845 \times 10^{-3} = 2,265\ kN/m$

$F_A = F_B - W_s - W_o = (3,994 - 1,707 - 2,265)\,10^3 = 22,19\ N/m$

$F_{TA} = F_A \times \ell = 22,19 \times 3 = 66,569\ N$ (for 3 m length)

Exercise 5.2

SB page 85

1.1 $A = \frac{\pi D^2}{4} = \frac{\pi \times 0,015^2}{4} = 176,715 \times 10^{-6}\ m^2$

$F_p = p \times A_p = 600 \times 10^3 \times 176,715 \times 10^{-6} = 106,029\ N$

1.2 Take moments about H:

$F_p \times y = F_B \times x 106,029 \times 0,03 = F_B \times 0,21\ F_B = 15,147\ N$

1.3 $V_w = \frac{F_B}{\rho g} = \frac{15,147}{1\,000 \times 9,81} = 1,544 \times 10^{-3}\ m^3$

$V_s = \frac{\pi D^3}{6} = \frac{\pi \times 0,2^3}{6} = 4,189 \times 10^{-3}\ m^3$

$\%V = \frac{V_W}{V_s} = \frac{1,544 \times 10^{-3}}{4,189 \times 10^{-3}} = 0,369 = 36,861\%$

2.1 $V_w = \frac{\pi \times D^2 \times \ell}{4} = \frac{\pi \times 0,8^2 \times 1,5}{4} = 753,982 \times 10^{-3}\ m^3$

$F_B = \rho g V_w = 10^3 \times 9,81 \times 753,982 \times 10^{-3} = 7,397\ kN$

2.2 $A_p = \frac{\pi D^2}{4} = \frac{\pi \times 0{,}8^2}{4} = 502{,}655 \times 10^{-3}\ m^2$

$h_c = \frac{D}{2} = \frac{0{,}8}{2} = 0{,}4\ m$

$F_w = \rho g h_c A = 10^3 \times 9{,}81 \times 0{,}2 \times 502{,}655 \times 10^{-3} = 1{,}972\ kN$

2.3 $F_R = \sqrt{F_B^2 + F_w^2} = \sqrt{7{,}397^2 + 1{,}972^2} = 7{,}655\ kN$

2.4 $\theta = \tan^{-1}\left(\frac{F_B}{F_w}\right) = \tan^{-1}\left(\frac{7{,}397}{1{,}972}\right) = 75{,}069°$

Exercise 5.3

SB page 93

1.1 $V_w = \frac{F_B}{\rho g} = \frac{100 \times 10^3}{10^3 \times 9{,}81} = 10{,}184\ m^3$

$d = \frac{V_w}{A_w} = \frac{10{,}184}{2{,}5 \times 4} = 1{,}019\ m$

1.2 Distance of centre of gravity from bottom

$y_g = \frac{h}{2} = \frac{1{,}9}{2} = 0{,}95\ m$

Distance of centre of buoyancy from bottom

$y_b = \frac{d}{2} = \frac{1{,}019}{2} = 0{,}51\ m$

Distance of metacentre from bottom

$I_s = \frac{\ell b^3}{12} = \frac{4 \times 2{,}5^3}{12} = 5{,}208\ m^4$

$MB = \frac{I_s}{V_w} = \frac{5{,}208}{10{,}184} = 0{,}511\ m$

$y_m = y_b + MB = 0{,}51 + 0{,}511 = 1{,}021\ m$

1.3 $MG = y_m - y_g = 1{,}021 - 0{,}95 = 70{,}621\ mm$ (above G)

The barge is stable because $y_m > y_g$

2.1 $V_o = \frac{F_B}{\rho g} = \frac{750}{810 \times 9{,}81} = 94{,}386 \times 10^{-3}\ m^3$

$h = \frac{V_o}{A_o} = \frac{94{,}386 \times 10^{-3}}{\pi \times 0{,}225^2} = 0{,}593\ m$

2.2 Distance of centre of gravity from bottom

$y_g = \frac{\ell}{2} = \frac{0{,}75}{2} = 0{,}375\ m$

Distance of centre of buoyancy from bottom

$y_b = \frac{h}{2} = \frac{0{,}593}{2} = 0{,}297\ m$

Distance of metacentre from bottom

$I_s = \frac{\pi d^4}{64} = \frac{\pi \times 0{,}45^4}{64} = 2{,}013 \times 10^{-3}\ m^4$

$MB = \frac{I_s}{V_o} = \frac{2{,}013 \times 10^{-3}}{94{,}386 \times 10^{-3}} = 21{,}326\ mm$

$y_m = y_b + MB = 0{,}297 + 21{,}326 \times 10^{-3} = 0{,}318\ m$

2.3 $MG = y_m - y_g = 0{,}318 - 0{,}375 = -56{,}943\ mm$ (below G)

The cylinder is unstable because $y_m < y_g$

3.1 $V_w = \frac{F_B}{\rho g} = \frac{100 \times 10^3}{10^3 \times 9{,}81} = 10{,}194\ m^3$

$d = \frac{V_w}{A_w} = \frac{10{,}194}{3 \times 5} = 0{,}68\ m$

Distance of centre of gravity from bottom

$y_g = \frac{h}{2} = \frac{1}{2} = 0{,}5\ m$

Distance of centre of buoyancy from bottom

$y_b = \frac{d}{2} = \frac{0{,}68}{2} = 0{,}34\ m$

Distance of metacentre from bottom

$I_s = \frac{\ell b^3}{12} = \frac{5 \times 3^3}{12} = 11{,}25\ m^4$

$MB = \frac{I_s}{V_w} = \frac{11{,}25}{10{,}194} = 1{,}104\ m$

$y_m = y_b + MB = 0{,}34 + 1{,}104 = 1{,}443\ m$

$MG = y_m - y_g = 1{,}443 - 0{,}5 = 0{,}943\ m$ (above G)

The barge is stable because $y_m > y_g$

3.2 $HG = MG\sin\theta = 0{,}943 \times \sin 6 = 98{,}614\ mm$

$M_R = HG \times W = 98{,}614 \times 10^{-3} \times 100 \times 10^3 = 9{,}861\ kNm$

Summative assessment: Module 5

SB page 96

1. The tendency of an immersed body to be lifted up in the fluid due to an upward force opposite to the action of gravity is known as buoyancy. ✓ (2)
2. It will sink if the density of the body is more than the fluid's density. ✓

   It will float if the density of the body is less than the fluid's density. ✓

   It will remain at rest if the density of the body is equal to the fluid's density. ✓ (3)

3. Stability refers to the ability of a body in a fluid to return to its original position ✓ after being tilted about a horizontal axis. ✓ (2)

4. Stable – The buoyant force causes a restoring moment which will bring the floating object back to its original position if displaced. ✓

   Neutral – The centre of gravity and the centre of buoyancy coincide. ✓

   Unstable – The buoyant force causes an overturning moment which will cause the floating object to topple over. ✓ (3)

5. Completely submerged bodies are stable if the centre of gravity is below the centre of buoyancy. ✓

   Floating objects are stable if the metacentre (M) is above the centre of gravity (G). The metacentre is the point at which the line of action of FB for the displaced position cuts the original vertical through the centre of gravity of the body. ✓

   Floating objects are unstable if the metacentre (M) is below the centre of gravity (G). ✓ (3)

6. When a body is immersed in a fluid either wholly or partially, ✓ it is lifted up by a force which is equal to the weight of the fluid displaced by the body ✓ and it acts upwards through the centroid of the displaced volume. ✓ (3)

7. The upward force exerted on a body ✓ immersed in a fluid which acts through the centroid of the displaced volume. ✓ (2)

8. The resultant force acting on a floating body by considering both the buoyant force ✓ and the hydrostatic force. ✓ (2)

9. $F = W + F_B$

   $F = \rho g V_w + \rho g V_w$

   $941{,}76 = 600 \times 9{,}81 \times V_w + 900 \times 9{,}81 \times V_w$ ✓

   $V_w = 64 \times 10^{-3}\ m^3$ ✓

   $\ell = \sqrt[3]{V_w} = \sqrt[3]{64 \times 10^{-3}} = 400\ mm$ ✓ (3)

10.1 $W_T = W_c + W_s = 500 + 200 = 700\ N$ ✓

   $V_W = \frac{F_B}{\rho_w g} = \frac{700}{1\,000 \times 9{,}81} = 71{,}356 \times 10^{-3}\ m^3$ ✓

   $d = \frac{V_W}{A} = \frac{71{,}356 \times 10^{-3}}{0{,}6 \times 0{,}9} = 132{,}14\ mm$ ✓ (3)

10.2 $V_s = \frac{W_s}{\rho_s g} = \frac{200}{7\,800 \times 9{,}81} = 2{,}614 \times 10^{-3}\ m^3$ ✓

$W_T = F_{BC} + F_{BS}$

$= \rho g V_c + \rho g V_s$

$700 = 1\,000 \times 9{,}81 \times V_c + 1\,000 \times 9{,}81 \times 2{,}614 \times 10^{-3}$ ✓

$V_c = 68{,}742 \times 10^{-3}\ m^3$ ✓

$d = \frac{V_c}{A} = \frac{68{,}742 \times 10^{-3}}{0{,}6 \times 0{,}9} = 127{,}3\ mm$ ✓ (4)

11.1 $V_w = \frac{\pi \times D^2 \times \ell}{4} = \frac{\pi \times 0{,}62^2 \times}{4} = 1{,}51\ m^3$ ✓

$F_B = \rho g V_w = 1\,030 \times 9{,}81 \times 1{,}51 = 15{,}253\ kN$ ✓ (2)

11.2 $V_s = \frac{\pi \times (D^2 - d^2) \times \ell}{4} = \frac{\pi \times (0{,}62^2 - 0{,}6^2) \times 5}{4} = 95{,}819 \times 10^{-3}\ m^3$ ✓

$W_s = \rho g V_s = 7\,800 \times 9{,}81 \times 95{,}819 \times 10^{-3} = 7{,}332\ kN$ ✓

$V_o = \frac{\pi \times d^2 \times \ell}{4} = \frac{\pi \times 0{,}6^2 \times 1}{4} = 1{,}414\ m^3$ ✓

$W_o = \rho g V_o = 800 \times 9{,}81 \times 1{,}414 = 11{,}095\ kN$ ✓

$F_A = W_s + W_o - F_B - =(7{,}332 - +11{,}095 - 15{,}253)\,10^3 = 3{,}174\ kN$ ✓ (5)

12.1 $A = \frac{\pi D^2}{4} = \frac{\pi \times 0{,}013^2}{4} = 132{,}732 \times 10^{-6}\ m^2$ ✓

$F_p = p \times A_p = 500 \times 10^3 \times 132{,}732 \times 10^{-6} = 66{,}366\ N$ ✓ (2)

12.2 Take moments about H:

$F_B \times x = F_p \times y$

$F_B \times 0{,}18 = 6{,}366 \times 0{,}03$ ✓

$F_B = 11{,}061\ N$ ✓ (2)

12.3 $V_w = \frac{F_B}{\rho g} = \frac{11{,}061}{1\,000 \times 9{,}81} = 1{,}128 \times 10^{-3}\ m^3$ ✓

$V_s = \frac{V_w}{\%} = \frac{1{,}128 \times 10^{-3}}{0{,}3} = 3{,}222 \times 10^{-3}\ m^3$ ✓

$D = \sqrt[3]{\frac{6 V_s}{\pi}} = \sqrt[3]{\frac{6 \times 3{,}222 \times 10^{-3}}{\pi}} = 183{,}24\ mm$ ✓ (3)

13.1 $V_w = A \times \ell = \left(3 \times 1 + \frac{3 \times 3}{2}\right) \times 4 = 30\ m^3$ ✓

$F_B = \rho g V_w = 1\ 030 \times 9{,}81 \times 30 = 303{,}129\ kN$ ✓ (2)

13.2 $A_p = (y + h)\ell = (1 + 3)4 = 16\ m^2$ ✓

$h_c = \frac{y + h}{2} = \frac{1 + 3}{2} = 2\ m$ ✓

$F_w = \rho g h_c A_p = 1\ 030 \times 9{,}81 \times 2 \times 16 = 323{,}338\ kN$ ✓ (3)

13.3 $F_R = \sqrt{F_B^2 + F_w^2} = \sqrt{303{,}129^2 + 323{,}338^2} = 443{,}209\ kN$ ✓ (1)

13.4 $\theta = \tan^{-1}\left(\frac{F_B}{F_w}\right) = \tan^{-1}\left(\frac{303{,}129}{323{,}338}\right) = 43{,}152°$ ✓ (1)

14.1 $V_w = \frac{F_B}{\rho g} = \frac{1\ 177{,}2 \times 10^3}{10^3 \times 9{,}81} = 120\ m^3$ ✓

$d = \frac{V_w}{A_w} = \frac{120}{8 \times 5} = 3\ m$ ✓

Distance of centre of gravity from bottom

$y_g = 4 - 2{,}2 = 1{,}8\ m$ ✓

Distance of centre of buoyancy from bottom

$y_b = \frac{d}{2} = \frac{3}{2} = 1{,}5\ m$ ✓

Distance of metacentre from bottom

$I_s = \frac{\ell b^3}{12} = \frac{8 \times 5^3}{12} = 83{,}333\ m^4$ ✓

$MB = \frac{I_s}{V_w} = \frac{83{,}333}{120} = 0{,}694\ m$ ✓

$y_m = y_b + MB = 1{,}5 + 0{,}694 = 2{,}194\ m$ ✓

$MG = y_m - y_g = 2{,}194 - 1{,}8 = 0{,}394\ m$ (above G) ✓ (8)

14.2 $\theta = \sin^{-1}\left(\frac{M_R}{W \times MG}\right) = \sin^{-1}\left(\frac{100 \times 10^3}{1\ 177{,}2 \times 10^3 \times 0{,}394}\right) = 12{,}437°$ ✓ (1)

Total: 60 marks

6 Fluid in motion

By the end of this module, students should be able to:

- describe flow patterns and Reynolds number;
- measure fluid flow using a rotameter;
- perform calculations for:
  - volumetric flow
  - mass flow
  - weight flow
  - the conservation of mass and velocity flow applied to round pipes; and
  - apply Bernoulli's equation.

Introduction

Fluid flow is a part of fluid mechanics that deals with the dynamics of fluids. The motion of a fluid subjected to unbalanced forces is known as *fluid dynamics*. As long as unbalanced pressures are applied, this motion will continue. Here we take a closer look at the various types of flow patterns and fluid flow.

Exercise 6.1

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1.1 $R_e = \frac{\rho vd}{\mu} = \frac{800 \times 2 \times 40 \times 10^{-3}}{45 \times 10^{-3}} = 1\ 422{,}222$ (laminar)

1.2 $R_e = \frac{\rho vd}{\mu} = \frac{800 \times 4 \times 40 \times 10^{-3}}{45 \times 10^{-3}} = 2\ 844{,}444$ (transition)

1.3 $R_e = \frac{\rho vd}{\mu} = \frac{800 \times 6 \times 40 \times 10^{-3}}{45 \times 10^{-3}} = 4\ 266{,}667$ (turbulant)

2.1 $Q = Av = \frac{\pi \times 0{,}03^2 \times 8}{4} = 5{,}655 \times 10^{-3}\ m^3/s$

2.2 $\dot{m} = \rho Q = 10^3 \times 5{,}655 \times 10^{-3} = 5{,}655$ kg/s

2.3 $\dot{w} = \dot{m}g = 5{,}655 \times 9{,}81 = 55{,}474$ N/s

2.4 $t = \frac{V}{Q} = \frac{800 \times 10^{-3}}{5{,}655 \times 10^{-3}} = 141{,}471$ sec

3.1 $Q_1 = A_1 v_1 = \frac{\pi \times 0{,}1^2 \times 4{,}5}{4} = 35{,}343 \times 10^{-3}\ m^3/s$

$v_2 = \frac{Q_2}{A_2} = \frac{35{,}343 \times 10^{-3}}{\frac{\pi \times 0{,}3^2}{4}} = 0{,}5\ m/s$

3.2 $\dot{m}_1 = \rho_1 Q_1 g = 850 \times 35{,}343 \times 10^{-3} \times 9{,}81 = 294{,}707\ N/s$

4.1 $Q_1 = A_1 v_1 = \frac{\pi \times 0{,}5^2 \times 2}{4} = 392{,}699 \times 10^{-3}\ m^3/s$

4.2 $Q_2 = A_2 v_2 = \frac{\pi \times 0{,}2^2 \times 8}{4} = 251{,}327 \times 10^{-3}\ m^3/s$

4.3 $Q_3 = Q_1 - Q_2 = 392{,}699 \times 10^{-3} - 251{,}327 \times 10^{-3} = 141{,}372 \times 10^{-3}\ m^3/s$

$v_3 = \frac{Q_3}{A_3} = \frac{141{,}372 \times 10^{-3}}{\frac{\pi \times 0{,}4^2}{4}} = 1{,}125\ m/s$

Exercise 6.2

SB page 113

1.1 $v = \frac{Q}{A} = \frac{0{,}5}{\frac{\pi \times 0{,}2^2}{4}} = 15{,}915\ m/s$

1.2 $E = \frac{p}{\rho g} + \frac{v^2}{2g} + z = \frac{200 \times 10^3}{10^3 \times 9{,}81} + \frac{15{,}915^2}{2 \times 9{,}81} + 5 = 26{,}199\ m$

2.1 $Q_1 = A_1 v_1 = \frac{\pi \times 02 \times 15}{4} = 4{,}712 \times 10^{-3}\ m^3/s$

2.2 $\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2 0 + \frac{15^2}{2 \times 9{,}81} + 0 = 0 + \frac{v_2^2}{2 \times 9{,}81} + 8 v_2 = 8{,}249\ m/s$

2.3 $A_2 = \frac{Q}{v_2} = \frac{4{,}712 \times 10^{-3}}{8{,}429} = 571{,}293 \times 10^{-3}\ m^2$

$d_2 = \sqrt{\frac{4 A_2}{\pi}} = \sqrt{\frac{4 \times 571{,}293 \times 10^{-3}}{\pi}} = 26{,}97\ mm$

2.4 $\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2 0 + \frac{15^2}{2 \times 9{,}81} + 0 = 0 + 0 + z_2 z_2 = 11{,}468\ m$

3.1 $v_1 = \frac{Q_1}{A_1} = \frac{10 \times 10^{-3}}{\frac{\pi \times 0{,}04^2}{4}} = 7{,}958\ m/s$

$v_2 = \frac{Q_2}{A_2} = \frac{10 \times 10^{-3}}{\frac{\pi \times 0{,}02^2}{4}} = 31{,}831\ m/s$

$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2 \frac{600 \times 10^3}{10^3 \times 9{,}81} + \frac{7{,}958^2}{2 \times 9{,}81} + 0 = \frac{p_2}{10^3 \times 9{,}81} + \frac{31{,}831^2}{2 \times 9{,}81} + 0$

$p_2 = 125{,}057\ kPa$

3.2 $\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2 \frac{600 \times 10^3}{10^3 \times 9{,}81} + \frac{7{,}958^2}{2 \times 9{,}81} + 0 = \frac{p_2}{10^3 \times 9{,}81} + \frac{31{,}831^2}{2 \times 9{,}81} + 4$

$p_2 = 85{,}817\ kPa$

Summative assessment: Module 6

SB page 116

1.

	Laminar	Turbulent
1	Fluid particles move in layers ✓	Fluid particles move in zig-zag manner ✓
2	Intermixing of fluid particles does not happen ✓	Intermixing of fluid particles does happen ✓
3	Occurs in liquids with high viscosity ✓	Occurs in liquids with low viscosity ✓
4	Reynolds number is less than 2 000 ✓	Reynolds number is more than 4 000 ✓
5	Laminar flow is rare in engineering ✓	Turbulent flow more frequent in engineering ✓

(5)

2.1 $Q = Av = \frac{\pi \times 0{,}05^2 \times 8}{4} = 15{,}709 \times 10^{-3}\ m^3/s = 15{,}709\ \ell/s$ ✓ (1)

2.2 $\dot{m} = \rho Q = 800 \times 15{,}709 \times 10^{-3} = 12{,}566\ kg/s$ ✓ (1)

2.3 $\dot{w} = \dot{m}g = 12{,}566 \times 9{,}81 = 123{,}276\ N/s$ ✓ (1)

2.4 $t = \frac{V}{Q} = \frac{900 \times 10^{-3}}{15{,}798 \times 10^{-3}} = 57{,}296\ sec$ ✓ (1)

2.5 $R_e = \frac{\rho v d}{\mu} = \frac{900 \times 8 \times 50 \times 10^{-3}}{45 \times 10^{-3}} = 7\ 111{,}111$ ✓ (turbulant) ✓ (2)

3.1 $v_1 = \frac{Q_1}{A_1} = \frac{4}{\frac{\pi \times 1{,}5^2}{4}} = 2{,}264\ m/s$ ✓ (2)

3.2 $Q_2 = 0{,}25\,Q_1 = 0{,}25 \times 4 = 1\ m^3/s$ ✓

$v_2 = \frac{Q_2}{A_2} = \frac{1}{\frac{\pi \times 0{,}8^2}{4}} = 1{,}989\ m/s$ ✓ (2)

3.3 $Q_3 = 0{,}75\,Q_1 = 0{,}75 \times 4 = 3\ m^3/s$

$A_3 = \frac{Q_3}{v_3} = \frac{3}{3} = 1\ m^2$ ✓

$d_3 = \sqrt{\frac{4A_3}{\pi}} = \sqrt{\frac{4 \times 1}{\pi}} = 1{,}128\ m$ ✓ (2)

4.1 $h = \frac{p}{\rho g} = \frac{200 \times 10^3}{900 \times 9{,}81} = 22{,}653 \text{ m}$ ✓ (1)

4.2 $\frac{p}{\rho g} + \frac{v^2}{2g} + z = E$

$22{,}653 + \frac{v^2}{2 \times 9{,}81} + 5 = 30$ ✓

$v = 6{,}786 \text{ m/s}$ ✓ (2)

4.3 $Q = Av = \frac{\pi \times 0{,}15^2 \times 6{,}786}{4} = 119{,}926 \times 10^{-3} \text{ m}^3\text{/s}$ ✓ $= 119{,}926\ \ell\text{/s}$ ✓ (2)

5.1 $\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2$

$0 + 0 + 3 = 0 + \frac{v_2^2}{2 \times 9{,}81} + 0$ ✓

$v_2 = 7{,}672 \text{ m/s}$ ✓ (2)

5.2 $Q = Av = \frac{\pi \times 0{,}1^2 \times 7{,}672}{4} = 60{,}265 \times 10^{-3} \text{ m}^3\text{/s}$ ✓

$\dot{m} = \rho Q = 10^3 \times 60{,}265 \times 10^{-3} = 60{,}265 \text{ kg/s}$ ✓ (2)

6.1 $v_x = \frac{Q_x}{A_x} = \frac{200 \times 10^{-3}}{\frac{\pi \times 0{,}6^2}{4}} = 0{,}707 \text{ m/s}$ ✓

$E_x = \frac{p_x}{\rho g} + \frac{v_x^2}{2g} + z_x$

$= \frac{80 \times 10^3}{10^3 \times 9{,}81} + \frac{0{,}707^2}{2 \times 9{,}81} + 5$ ✓

$E_x = 13{,}18 \text{ m}$ ✓ (3)

6.2 $v_x = \frac{Q_x}{A_x} = \frac{200 \times 10^{-3}}{\frac{\pi \times 0{,}3^2}{4}} = 2{,}829 \text{ m/s}$ ✓

$\frac{p_y}{\rho g} + \frac{v_y^2}{2g} + z_y = E\ \frac{100 \times 10^3}{10^3 \times 9{,}81} + \frac{2{,}829^2}{2 \times 9{,}81} + z_y = 13{,}18$ ✓

$z_y = 2{,}579 \text{ m}$ ✓ (3)

7.1 $v_1 = \frac{Q_1}{A_1} = \frac{900 \times 10^{-3}}{\frac{\pi \times 1^2}{4}} = 1{,}146 \text{ m/s}$ ✓

$E_1 = \frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1$

$= \frac{75 \times 10^3}{10^3 \times 9{,}81} + \frac{1{,}146}{2 \times 9{,}81} + 3$ ✓

$E_1 = 10{,}712 \text{ m}$ ✓ (3)

7.2 $v_2 = \frac{Q_2}{A_2} = \frac{900 \times 10^{-3}}{\frac{\pi \times 0,6^2}{4}} = 3,183 \text{ m/s}$ ✓

$E_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2$

$10,712 = \frac{p_2}{10^3 \times 9,81} + \frac{3,183^2}{2 \times 9,81} + 0$ ✓

$p_2 = 100,021 \text{ kPa}$ ✓ (3)

8.1 $A_2 v_2 = A_1 v_1 \frac{\pi \times 0,3^2 \times v_2}{4} = \frac{\pi \times 0,3^2 \times v_1}{4}$

$v_2 = 9 v_1 - - - (1)$

$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2$

$\frac{90 \times 10^3}{10^3 \times 9,81} + \frac{v_1^2}{2 \times 9,81} + 9 = \frac{130 \times 10^3}{10^3 \times 9,81} + \frac{(9 v_1)^2}{2 \times 9,81} + 0$ ✓

$v_1 = 1,099 \text{ m/s}$ ✓ (4)

8.2 $v_2 = 9 v_1 = 9 \times 1,099 = 9,889 \text{ m/s}$ ✓ (1)

8.3 $Q_1 = A_1 v_1 = \frac{\pi \times 0,9^2 \times 1,099}{4} = 698,994 \times 10^{-3} \text{ m}^3/\text{s}$ ✓

$\dot{m}_1 = \rho_1 Q_1 = 10^3 \times 698,994 \times 10^{-3} = 698,994 \text{ kg/s}$ ✓ (2)

Total: 45 marks

MODULE

7 Flow measurement using instruments

By the end of this module, students should be able to:

- calculate and explain the general applications of Bernoulli's equation on flow measurement in the following instruments:
  - venturi meter
  - pitot static tube
  - manometer.

Introduction

Measuring the flow of liquids is a critical need in many industrial applications. In some operations, the ability to conduct accurate flow measurements is so important that it can make the difference between making a profit or taking a loss.

Flow measurement involves using flow meters that measure the amount of fluid moving through a pipe, channel or space by measuring volumetric flow rates. Measuring the flow rate of liquids is necessary to control as well as check the quality of industrial processes. It assures the exactness in fluid quantity and quality. Flow measurement also helps to determine the functioning of a system. For example, if there is a leak in a system, the quantity of the fluid will not be accurate and will be indicated by the flowmeter.

In this module, we will be discussing everything you need to know about flow measurement and its application in the industry, including its principle, the devices used for flow measurement, and the different methods of flow measurement. There are many different types of flow meters that can be utilised, depending on the nature of the application.

Exercise 7.1

SB page 124

1.1 $v = C_v\sqrt{2gh} = 0,94\sqrt{2 \times 9,81 \times 0,09} = 1,249\ \text{m/s}$

1.2 $Q = Av = \frac{\pi \times 0,075^2 \times 1,249}{4} = 5,518 \times 10^{-3}\ \text{m}^3/\text{s}$

$\dot{m} = \rho Q = 800 \times 5,518 \times 10^{-3} = 4,415\ \text{kg/s}$

2.1 $h = h_m\left[\frac{\rho_m}{\rho_\ell} - 1\right] = 0,07\left[\frac{1\,000}{1,2} - 1\right] = 58,263\ \text{m}$

$v = C_v\sqrt{2gh} = 0,93\sqrt{2 \times 9,81 \times 58,263} = 31,443\ \text{m/s}$

2.2 $Q = Av = \frac{\pi \times 0,04^2 \times 31,443}{4} = 39,513 \times 10^{-3}\ \text{m}^3/\text{s} = 39,513\ \ell/\text{s}$

3.1 $v = \frac{\dot{m}}{\rho A} = \frac{15}{800 \times \pi \times 0,06^2} = 1,658\ \text{m/s}$

3.2 $v_a = C_v\sqrt{2gh}\ 1,658 = 0,98\sqrt{2 \times 9,81 \times h}\ h = 145,864\ \text{mm}$

$h_m = \frac{h}{\left[\frac{\rho_m}{\rho_\ell} - 1\right]} = \frac{145,864}{\left[\frac{13\,600}{800} - 1\right]} = 9,116\ \text{mm}$

Exercise 7.2

SB page 132

1.1 $v_1 = \frac{Q}{A_1} = \frac{10 \times 10^{-3}}{\pi \times 0,045^2} = 1,572\ \text{m/s}$

$v_2 = \frac{Q}{A_2} = \frac{10 \times 10^{-3}}{\pi \times 0,015^2} = 14,147\ \text{m/s}$

1.2 $\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2$

$\frac{p_1}{10^3 \times 9,81} + \frac{1,572^2}{2 \times 9,81} + 0 = \frac{90 \times 10^3}{10^3 \times 9,81} + \frac{14,147^2}{2 \times 9,81} + 0$

$p_2 = 188,835\ \text{kPa}$

2.1 $A_2 v_2 = A_1 v_1 \frac{\pi \times 0,04^2 \times v_2}{4} = \frac{\pi \times 0,09^2 \times v_1}{4} v_2 = 5,0625\, v_1 - - - (1)$

2.2 $h = \frac{p_1 - p_2}{\rho g} = \frac{400 \times 10^3 - 350 \times 10^3}{10^3 \times 9,81} = 5,097\ \text{m}$

2.3 $\frac{v_2^2 - v_1^2}{2g} = h \frac{(5,0625\, v_1)^2 - v_1^2}{2 \times 9,81} = 5,097\, v_1 = 2,015\ \text{m/s}$

$Q = C_d A_1 v_1 = 0,96 \times \pi \times 0,045^2 \times 2,105 = 112,306 \times 10^{-3}\ \text{m}^3/\text{s}$

3.1 $v_1 = \frac{Q}{A_1} = \frac{30 \times 10^{-3}}{\pi \times 0{,}075^2} = 1{,}698 \text{ m/s}$

$v_2 = \frac{Q}{A_2} = \frac{30 \times 10^{-3}}{\pi \times 0{,}0375^2} = 6{,}791 \text{ m/s}$

$h = \frac{v_2^2 - v_1^2}{2g} = \frac{6{,}791^2 - 1{,}698^2}{2 \times 9{,}81} = 2{,}203 \text{ m}$

3.2 $h_m = \frac{h}{\left[\frac{\rho_m}{\rho_\ell} - 1\right]} = \frac{2{,}203}{\left[\frac{13\,600}{1\,000} - 1\right]} = 174{,}872 \text{ mm}$

4.1 $v_2 = \frac{A_1}{A_2} v_1 = \frac{450^2}{150^2} = 9\,v_1$

4.2 $h = h_m\left[\frac{\rho_m}{\rho_\ell} - 1\right] = 0{,}4\left[\frac{13\,600}{750} - 1\right] = 6{,}853 \text{ m}$

4.3 $\frac{v_2^2 - v_1^2}{2g} + z_2 = h + z_1 \frac{(9\,v_1)^2 - v_1^2}{2 \times 9{,}81} + 0{,}9 = 6{,}853 + 0\,v_1 = 1{,}208 \text{ m/s}$

$Q = C_d A_1 v_1 = 0{,}95 \times \pi \times 0{,}225^2 \times 1{,}208 = 182{,}567 \times 10^{-3} \text{ m}^3/\text{s}$

$\dot{m} = \rho Q = 750 \times 182{,}567 \times 10^{-3} = 136{,}926 \text{ kg/s}$

Summative assessment: Module 7

SB page 134

1.1 $h = h_2 - h_1 = 4 - 2{,}6 = 1{,}4 \text{ m}$ ✓

$v = C_v\sqrt{2gh} = 0{,}98\sqrt{2 \times 9{,}81 \times 1{,}4} = 5{,}136 \text{ m/s}$ ✓ (2)

1.2 $Q = Av = \pi \times 0{,}045^2 \times 5{,}136 = 32{,}675 \times 10^{-3} \text{ m}^3/\text{s}$ ✓

$\dot{m} = \rho Q = 1\,000 \times 32{,}675 \times 10^{-3} = 32{,}675 \text{ kg/s}$ ✓ (2)

2.1 $h = \frac{p_1 - p_2}{\rho g} = \frac{12 \times 10^3 - (-16 \times 10^3)}{800 \times 9{,}81} = 3{,}568 \text{ m}$ ✓

$v = C_v\sqrt{2gh} = 0{,}98\sqrt{2 \times 9{,}81 \times 3{,}568} = 8{,}199 \text{ m/s}$ ✓ (2)

2.2 $Q = Av = \pi \times 0{,}2^2 \times 8{,}199 = 1{,}03 \text{ m}^3/\text{s}$ ✓

$\dot{m} = \rho Q = 800 \times 1{,}03 = 824{,}28 \text{ kg/s}$ ✓ (2)

3.1 $h = h_m\left[\frac{\rho_m}{\rho_\ell} - 1\right] = 0{,}2\left[\frac{13\,600}{1025} - 1\right]$ ✓ $= 2{,}454 \text{ m}$ ✓ (2)

3.2 $v = C_v\sqrt{2gh} = 0{,}97\sqrt{2 \times 9{,}81 \times 2{,}454} = 6{,}73 \text{ m/s}$ ✓ (2)

4.1 $v_2 = \frac{A_1}{A_2} v_1 = \frac{200^2}{100^2} = 4\,v_1$ ✓ (1)

4.2 $h = \frac{p_1 - p_2}{\rho g} = \frac{180 \times 10^3 - (-35 \times 10^3)}{10^3 \times 9{,}81}$ ✓ $= 21{,}916 \text{ m}$ ✓ (2)

4.3 $\frac{v_2^2 - v_1^2}{2g} = h$

$\frac{(4v_1)^2 - v_1^2}{2 \times 9{,}81} = 21{,}916$ ✓

$v_1 = 5{,}354$ m/s ✓

$Q = C_d A_1 v_1 = 0{,}96 \times \pi \times 0{,}1^2 \times 5{,}354 = 161{,}477 \times 10^{-3}\ m^3/s$ ✓

$\dot{m} = \rho Q = 1000 \times 161{,}477 \times 10^{-3} = 161{,}477$ kg/s ✓ (4)

5.1 $v_1 = \frac{Q}{A_1} = \frac{100 \times 10^{-3}}{\pi \times 0{,}1^2} = 3{,}183$ m/s ✓

$\frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2 = \frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1$

$0 + \frac{v_2^2}{2 \times 9{,}81} + 0 = \frac{50 \times 10^3}{10^3 \times 9{,}81} + \frac{3{,}183^2}{2 \times 9{,}81} + 0$ ✓

$v_2 = 10{,}494$ m/s ✓ (3)

5.2 $A_2 = \frac{Q}{v_2} = \frac{100 \times 10^{-3}}{10{,}494} = 9{,}529 \times 10^{-3}\ m^2$ ✓

$d = \sqrt{\frac{4A_2}{\pi}} = \sqrt{\frac{4 \times 9{,}529 \times 10^{-3}}{\pi}} = 110{,}148$ mm ✓ (2)

6.1 $v_2 = \frac{A_1}{A_2} v_1 = \frac{400^2}{200^2} = 4v_1$ (1)

6.2 $h = h_m\left[\frac{\rho_m}{\rho_\ell} - 1\right] = 0{,}06\left[\frac{13\,600}{900} - 1\right] = 0{,}847$ m ✓

$z_2 = \ell \sin\theta = 0{,}9 \sin 40 = 0{,}579$ m

$\frac{v_2^2 - v_1^2}{2g} + z_2 = h + z_1$

$\frac{(4v_1)^2 - v_1^2}{2 \times 9{,}81} + 0{,}579 = 0{,}847 + 0$ ✓

$v_1 = 0{,}592$ m/s ✓

$Q = C_d A_1 v_1 = 0{,}98 \times \pi \times 0{,}2^2 \times 0{,}592 = 72{,}935 \times 10^{-3}\ m^3/s$ ✓

$\dot{m} = \rho Q = 900 \times 71{,}935 \times 10^{-3} = 65{,}641$ kg/s ✓ (5)

Total: 30 marks

8 Pipeline systems

By the end of this module, students should be able to:

- perform calculations on shock losses, including the following:
  - sudden enlargement
  - sudden contraction
  - entry and exit losses of reservoirs, valves and bends in pipelines;
- perform calculations on friction losses in pipelines with the D'arcy formula; and
- equivalent length and equivalent ratios for open and closed tanks.

Introduction

In previous modules, it was assumed that when a fluid flows through a pipeline, no losses occur in the system. A small compensation was made when the flow velocity or flow rate was calculated by introducing a coefficient that was used to distinguish between theoretical flow rate and actual flow rate. The losses were small due to the fact that the pipe lengths were short and the changes in diameters were gradual, resulting in ignoring of these losses.

In big distribution networks, there are very long lengths of pipes and numerous apparatus in the pipe systems. The losses caused by long lengths of pipes and the existence of fittings, bends and valves in pipelines cannot be ignored when the pressure loss and power needed to drive the fluid, need to be calculated.

Exercise 8.1

SB page 144

1.1 $v_1 = \frac{Q}{A_1} = \frac{0,01}{\pi \times 0,015^2} = 14,147 \text{ m/s}$

$v_2 = \frac{Q}{A_2} = \frac{0,01}{\pi \times 0,03^2} = 3,537 \text{ m/s}$

$v_3 = \frac{Q}{A_3} = \frac{0,01}{\pi \times 0,02^2} = 7,958 \text{ m/s}$

Entrance of pipeline from reservoir

$h_{s1} = 0,5 \times \frac{v_1^2}{2g} = \frac{0,5 \times 14,147^2}{2 \times 9,81} = 5,1 \text{ m}$

Sudden enlargement

$$h_{s2} = \frac{(v_1 - v_2)^2}{2g} = \frac{(14{,}147 - 3{,}537)^2}{2 \times 9{,}81} = 5{,}738 \text{ m}$$

Sudden contraction

$$h_{s3} = \left(\frac{1}{C_c} - 1\right)^2 \times \frac{v_3^2}{2g} = \left(\frac{1}{0{,}6} - 1\right)^2 \times \frac{7{,}958^2}{2 \times 9{,}81} = 2{,}152 \text{ m}$$

$$h_L = h_{s1} + h_{s2} + h_{s3} = 5{,}1 + 5{,}738 + 2{,}152 = 12{,}99 \text{ m}$$

1.2 $p = \rho g h_L = 10^3 \times 9{,}81 \times 12{,}99 = 127{,}433 \text{ kPa}$

1.3 $P = pQ = 127{,}433 \times 10^3 \times 0{,}01 = 1{,}274 \text{ kW}$

2.1 $v = \frac{Q}{A} = \frac{0{,}08}{\pi \times 0{,}05^2} = 10{,}186 \text{ m/s}$

$$k_f = \frac{4fL}{d} = \frac{4 \times 0{,}015 \times 20}{0{,}1} = 12$$

$$h_f = k_f \times \frac{v^2}{2g} = 12 \times \frac{10{,}186^2}{2 \times 9{,}81} = 63{,}457 \text{ m}$$

2.2 $p = \rho g h_f = 850 \times 9{,}81 \times 63{,}457 = 529{,}14 \text{ kPa}$

2.3 $P = pQ = 529{,}14 \times 10^3 \times 0{,}08 = 42{,}331 \text{ kW}$

3.1 $k_f = \frac{4fL}{d} = \frac{4 \times 0{,}013 \times 30}{0{,}06} = 26$

$$k_T = k_f + k_v + 2 \times k_b = 26 + 1{,}4 + 2 \times 0{,}3 = 28$$

3.2 $L_e = \frac{k_L d}{4f} = \frac{28 \times 0{,}06}{4 \times 0{,}013} = 32{,}308 \text{ m}$

3.3 $v = \frac{Q}{A} = \frac{0{,}025}{\pi \times 0{,}03^2} = 8{,}842 \text{ m/s}$

$$h_L = k_T \times \frac{v^2}{2g} = 28 \times \frac{8{,}842^2}{2 \times 9{,}81} = 111{,}572 \text{ m}$$

3.4 $p = \rho g h_L = 10^3 \times 9{,}81 \times 111{,}572 = 1{,}095 \text{ MPa}$

Exercise 8.2

SB page 152

1.1 $h_L = \frac{4fL}{d} \times \frac{v_2^2}{2g} + 0,5 \times \frac{v_2^2}{2g}$

$= \frac{4 \times 0,01 \times 15}{0,03} \times \frac{v_2^2}{2g} + 0,5 \times \frac{v_2^2}{2g}$

$h_L = 20,5 \times \frac{v_2^2}{2g}$

$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2 + h_L$

$0 + 0 + z_1 = 0 + \frac{v_2^2}{2g} + 0 + h_L$

$25 = \frac{v_2^2}{2 \times 9,81} + 20,5 \times \frac{v_2^2}{2 \times 9,81}$

$v_2 = 4,776 \text{ m/s}$

1.2 $Q = Av = \pi \times 0,015^2 \times 4,776 = 3,376 \times 10^{-3} \text{ m}^3/\text{s}$

$\dot{m} = \rho Q = 10^3 \times 3,376 \times 10^{-3} = 3,376 \text{ kg/s}$

2.1 $h_L = \frac{4fL}{d} \times \frac{v^2}{2g} + 0,5 \times \frac{v^2}{2g} + \frac{v^2}{2g}$

$= \frac{4 \times 0,012 \times 35}{0,06} \times \frac{v^2}{2g} + 0,5 \times \frac{v^2}{2g} + \frac{v^2}{2g}$

$h_L = 29,5 \times \frac{v^2}{2g}$

$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2 + h_L$

$0 + 0 + z_1 = 0 + 0 + 0 + h_L$

$30 = 29,5 \times \frac{v^2}{2g}$

$v = 4,467 \text{ m/s}$

2.2 $Q = Av = \pi \times 0,03^2 \times 4,467 = 12,63 \times 10^{-3} \text{ m}^3/\text{s}$

$\dot{m} = \rho Q = 10^3 \times 12,63 \times 10^{-3} = 12,63 \text{ kg/s}$

3.1 $h_L = \frac{4fL}{d} \times \frac{v_2^2}{2g} + 0,5 \times \frac{v_2^2}{2g} + 4,5 \times \frac{v_2^2}{2g}$

$= \frac{4 \times 0,01 \times 40}{0,05} \times \frac{v_2^2}{2g} + 0,5 \times \frac{v_2^2}{2g} + 4,5 \times \frac{v_2^2}{2g}$

$h_L = 37 \times \frac{v_2^2}{2g}$

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2 + h_L$$

$$\frac{p_1}{\rho g} + 0 + z_1 = 0 + \frac{v_2^2}{2g} + 0 + h_L$$

$$\frac{120 \times 10^3}{10^3 \times 9{,}81} + 15 = \frac{v_2^2}{2g} + 37 \times \frac{v_2^2}{2g}$$

$$v_2 = 3{,}75 \text{ m/s}$$

3.2 $Q = Av = \pi \times 0{,}025^2 \times 3{,}75 = 7{,}363 \times 10^{-3} \text{ m}^3/\text{s}$

$\dot{m} = \rho Q = 10^3 \times 7{,}363 \times 10^{-3} = 7{,}363 \text{ kg/s}$

4.1 $v = \frac{Q}{A} = \frac{0{,}02}{\pi \times 0{,}045^2} = 3{,}144 \text{ m/s}$

4.2 $h_L = \frac{4fL}{d} \times \frac{v_2^2}{2g} + 0{,}5 \times \frac{v_2^2}{2g}$

$$= \frac{4 \times 0{,}012 \times 100}{0{,}09} \times \frac{3{,}144^2}{2 \times 9{,}81} + 0{,}5 \times \frac{3{,}144^2}{2 \times 9{,}81}$$

$$h_L = 27{,}118 \text{ m}$$

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2 + h_L \; 0 + 0 + z_1 = 0 + 0 + 0 + 27{,}118$$

$$z_1 = 27{,}118 \text{ m}$$

5.1 $v_1 = \frac{A_2}{A_1} \times v_2 = \frac{90^2}{30^2} \times v_2 = 9 v_2$

5.2 $h_L = 0{,}5 \times \frac{v_1^2}{2g} + \frac{(v_1 - v_2)^2}{2g} + \frac{v_2^2}{2g} + \frac{4fL}{d} \times \frac{v_1^2}{2g} + \frac{4fL}{d} \times \frac{v_2^2}{2g}$

$$= 0{,}5 \times \frac{(9 v_2)^2}{2g} + \frac{(9v_2 - v_2)^2}{2g} + \frac{v_2^2}{2g} + \frac{4fL}{d} \times \frac{(9 v_2)^2}{2g} + \frac{4fL}{d} \times \frac{v_2^2}{2g}$$

$$= 40{,}5 \times \frac{v_2^2}{2g} + 64 \times \frac{v_2^2}{2g} + \frac{v_2^2}{2g} + \frac{4 \times 0{,}01 \times 10}{0{,}03} \times \frac{81 v_2^2}{2g} + \frac{4 \times 0{,}01 \times 20}{0{,}09} \times \frac{v_2^2}{2g}$$

$$h_L = 1194{,}389 \times \frac{v_2^2}{2g}$$

$$\frac{p_A}{\rho g} + \frac{v_A^2}{2g} + z_A = \frac{p_B}{\rho g} + \frac{v_B^2}{2g} + z_B + h_L$$

$$0 + 0 + z_A = 0 + 0 + 0 + h_L$$

$$30 = 1194{,}389 \times \frac{v_2^2}{2g}$$

$$v_2 = 0{,}702 \text{ m/s}$$

5.3 $Q = A_2 v_2 = \pi \times 0{,}045^2 \times 0{,}702 = 4{,}466 \times 10^{-3} \text{ m}^3/\text{s}$

$\dot{m} = \rho Q = 10^3 \times 4{,}466 \times 10^{-3} = 4{,}466 \text{ kg/s}$

Summative assessment: Module 8 **SB page 156**

1.1 $v = \frac{Q}{A} = \frac{9 \times 10^{-3}}{\pi \times 0{,}015^2}$ ✓ $= 12{,}732$ m/s ✓ (2)

1.2 $h_L = \frac{4fL}{d} \times \frac{v_2^2}{2g} + 0{,}5 \times \frac{v_2^2}{2g}$

$= \frac{4 \times 0{,}01 \times 17}{0{,}03} \times \frac{12{,}732^2}{2 \times 9{,}81}$ ✓ $+ 0{,}5 \times \frac{12{,}732^2}{2 \times 9{,}81}$ ✓

$h_L = 191{,}419$ m ✓

$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2 + h_L$

$0 + 0 + z_1 = 0 + \frac{12{,}732^2}{2g} + 0 + 191{,}419$ ✓

$z_1 = 199{,}682$ m ✓ (5)

1.3 $h_L = k_L \times \frac{v^2}{2g} = 0{,}5 \times \frac{12{,}732^2}{2 \times 9{,}81} = 4{,}131$ m ✓

$p = \rho g h_v = 10^3 \times 9{,}81 \times 4{,}131 = 40{,}528$ kPa ✓ (2)

2.1 $h_L = H = 30$ m ✓ (1)

2.2 $\frac{4fL}{d} \times \frac{v_2^2}{2g} + 0{,}5 \times \frac{v_2^2}{2g} = h_L$

$\frac{4 \times 0{,}012 \times 35}{d} \times \frac{5^2}{2 \times 9{,}81}$ ✓ $+ 0{,}5 \times \frac{5^2}{2 \times 9{,}81}$ ✓ $= 30$

$d = 72{,}904$ mm ✓ (3)

2.3 $Q = Av = \pi \times 0{,}036^2 \times 5 = 20{,}872 \times 10^{-3}$ m^3/s ✓

$\dot{m} = \rho Q = 10^3 \times 20{,}872 \times 10^{-3} = 20{,}872$ kg/s ✓ (2)

3.1 $v = \frac{Q}{A} = \frac{7 \times 10^{-3}}{\pi \times 0{,}03^2}$ ✓ $= 2{,}476$ m/s ✓ (2)

3.2 $h_L = \frac{4fL}{d} \times \frac{v_2^2}{2g} + 0{,}5 \times \frac{v_2^2}{2g} + 0{,}5 \times \frac{v_2^2}{2g}$

$= \frac{4 \times 0{,}01 \times 40}{0{,}06} \times \frac{2{,}476^2}{2 \times 9{,}81}$ ✓ $+ 0{,}5 \times \frac{2{,}476^2}{2 \times 9{,}81} + 3{,}5 \times \frac{2{,}476^2}{2 \times 9{,}81}$ ✓

$h_L = 9{,}58$ m ✓ (3)

3.3 $\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2 + h_L$

$\frac{p_1}{10^3 \times 9{,}81} + 0 + 5$ ✓ $= 0 + \frac{2{,}476^2}{2 \times 9{,}81} + 0 + 9{,}58$ ✓

$p_1 = 47{,}997$ kPa ✓ (3)

4.1 $v = \frac{Q}{A} = \frac{20 \times 10^{-3}}{\pi \times 0{,}045^2}$ ✓ $= 3{,}144$ m/s ✓ (2)

4.2 $h_L = H = 30$ m ✓

$$\frac{4fL}{d} \times \frac{v_2^2}{2g} + 0{,}5 \times \frac{v_2^2}{2g} = h_L$$

$$\frac{4 \times 0{,}012 \times L}{0{,}09} \times \frac{3{,}144^2}{2 \times 9{,}81} \checkmark + 0{,}5 \times \frac{3{,}144^2}{2 \times 9{,}81} \checkmark = 30$$

$L = 110{,}726$ m ✓ (4)

4.3 $k_T = \frac{2g h_L}{v^2} = \frac{2 \times 9{,}81 \times 30}{3{,}144^2} = 59{,}554$ ✓

$\frac{L}{d} = \frac{k_T}{4f} = 1\,240{,}706$ ✓ (2)

5.1 $v_2 = \frac{A_1}{A_2} \times v_1 = \frac{40^2}{20^2} \times v_2 = 4v_1$ ✓ (1)

5.2 $h_L = H = 30$ m ✓

$$h_s = 0{,}5 \times \frac{v_1^2}{2g} + \left(\frac{1}{C_c} - 1\right)^2 \frac{v_2^2}{2g}$$

$$= 0{,}5 \times \frac{v_1^2}{2g} + \left(\frac{1}{0{,}6} - 1\right)^2 \frac{(4v_1)^2}{2g}$$

$h_s = 7{,}611 \frac{v_1^2}{2g}$ ✓

$$h_f = \frac{4fL}{d} \times \frac{v_1^2}{2g} + \frac{4fL}{d} \times \frac{v_2^2}{2g}$$

$$= \frac{4 \times 0{,}01 \times 5}{0{,}04} \times \frac{v_1^2}{2g} + \frac{4 \times 0{,}01 \times 10}{0{,}02} \times \frac{(4v_1)^2}{2g}$$

$h_f = 325 \frac{v_1^2}{2g}$ ✓

$h_L = h_s + h_f = 7{,}611 \frac{v_1^2}{2g} + 325 \frac{v_1^2}{2g} = 332{,}611 \frac{v_1^2}{2g}$ ✓

$332{,}611 \times \frac{v_1^2}{2 \times 9{,}81} = 30$ ✓

$v_1 = 1{,}33$ m/s ✓ (6)

5.3 $Q = A_1 v_1 = \pi \times 0{,}02^2 \times 1{,}33 = 1{,}672 \times 10^{-3}$ m³/s ✓

$\dot{m} = \rho Q = 800 \times 1{,}672 \times 10^{-3} = 1{,}337$ kg/s ✓ (2)

Total: 40 marks

9 Flow through orifices

By the end of this module, students should be able to:
- define the term orifice;
- define the term vena contracta;
- calculate flow through orifices using equations with different velocities and discharges;
- perform calculations on flow through different types of small orifices in the side of a reservoir under a certain head of water;
- calculate the three flow coefficients; and
- calculate the head loss in an orifice.

Introduction

An orifice is an opening, of any size or shape, in a pipe or at the bottom or side wall of a container through which fluid is discharged. This opening is typically created by placing a plate or a thin edge within a fluid-carrying pipe or container. The shape and size of the orifice plays a crucial role in determining the characteristics of fluid flow through it. If the geometric properties of the orifice and the properties of the fluid are known, the orifice can be used to measure flow rates.

Exercise 9.1

SB page 167

1.1 $A_a = \frac{Q_a}{v_a} = \frac{0,003}{7,8} = 384,615 \times 10^{-6}\ m^2$

$C_c = \frac{A_a}{A_t} = \frac{384,615 \times 10^{-6}}{500 \times 10^{-6}} = 0,769$

1.2 $v_t = \sqrt{2gh} = \sqrt{2 \times 9,81 \times 4} = 8,859\ m/s$

$C_v = \frac{v_a}{v_t} = \frac{7,8}{8,859} = 0,88$

1.3 $C_d = C_c \times C_v = 0,88 \times 0,769 = 0,677$

2.1 $A_a = \pi \times 0,0115^2 = 415,476 \times 10^{-6}\ m^2$

$A_t = \pi \times 0,0,0125^2 = 490,874 \times 10^{-6}\ m^2$

$$C_c = \frac{A_a}{A_t} = \frac{415,476 \times 10^{-6}}{490,874 \times 10^{-6}} = 0,846$$

2.2 $v_t = \sqrt{2gh} = \sqrt{2 \times 9,81 \times 50} = 31,321\ m/s$

$$v_a = \frac{Q_a}{A_a} = \frac{0,01}{415,476 \times 10^{-6}} = 24,069\ m/s$$

$$C_v = \frac{v_a}{v_t} = \frac{24,069}{31,321} = 0,768$$

2.3 $C_d = C_c \times C_v = 0,846 \times 0,768 = 0,65$

2.4 $h_L = h\left(1 - C_v^2\right) = 50\left(1 - 0,768^2\right) = 20,474\ m$

3.1 $A_t = \pi \times 0,0,01^2 = 314,159 \times 10^{-6}\ m^2$

$A_a = C_c \times A_t = 0,65 \times 314,159 \times 10^{-6} = 204,204 \times 10^{-6}\ m^2$

$$d = \sqrt{\frac{4A_a}{\pi}} = \sqrt{\frac{4 \times 204,204 \times 10^{-6}}{\pi}} = 16,125\ mm$$

3.2 $v_t = \sqrt{2gh} = \sqrt{2 \times 9,81 \times 20} = 19,809\ m/s$

$v_a = C_v \times v_t = 0,9 \times 19,809 = 17,828\ m/s$

3.3 $Q_a = A_a \times v_a = 204,204 \times 10^{-6} \times 17,828 = 3,641 \times 10^{-3}\ m^3/s$

$\dot{m} = \rho Q_a = 850 \times 3,641 \times 10^{-3} = 3,094\ kg/s$

3.4 $h_L = h\left(1 - C_v^2\right) = 20\left(1 - 0,65^2\right) = 11,55\ m$

$p = \rho g h_L = 850 \times 9,81 \times 11,55 = 96,31\ kPa$

4.1 $A_a = \pi \times 0,01^2 = 314,159 \times 10^{-6}\ m^2$

$$A_t = \frac{A_a}{C_c} = \frac{314,159 \times 10^{-6}}{0,84} = 373,999 \times 10^{-6}\ m^2$$

$$d = \sqrt{\frac{4A_t}{\pi}} = \sqrt{\frac{4 \times 632,059 \times 10^{-6}}{\pi}} = 21,822\ mm$$

4.2 $C_v = \frac{C_d}{C_c} = \frac{0,63}{0,84} = 0,75$

4.3 $v_a = \frac{Q_a}{A_a} = \frac{0,005}{314,159 \times 10^{-6}} = 15,915\ m/s$

$$v_t = \frac{v_a}{C_v} = \frac{15,915}{0,75} = 21,221\ m/s$$

$$h = \frac{v_t^2}{2g} = \frac{21,221^2}{2 \times 9,81} = 22,952\ m$$

5.1 $A_a = \pi \times 0,0,009^2 = 254,469 \times 10^{-6}\ m^2$

$A_t = \pi \times 0,0,01^2 = 314,159 \times 10^{-6}\ m^2$

$C_c = \frac{A_a}{A_t} = \frac{254,469 \times 10^{-6}}{314,159 \times 10^{-6}} = 0,81$

5.2 $C_v = \frac{C_d}{C_c} = \frac{0,6}{0,81} = 0,741$

5.3 $v_a = \frac{Q_a}{A_a} = \frac{0,003}{254,469 \times 10^{-6}} = 11,789\ m/s$

$v_t = \frac{v_a}{C_v} = \frac{11,789}{0,741} = 15,915\ m/s$

$\frac{p_A}{\rho g} + \frac{v_A^2}{2g} + z_A = \frac{p_B}{\rho g} + \frac{v_B^2}{2g} + z_B \frac{p_A}{750 \times 9,81} + 0 + 4 = 0 + \frac{15,915^2}{2 \times 9,81} + 0$

$p_A = 65,559\ kPa$

Summative assessment: Module 9 **SB page 170**

1. An orifice is an opening, of any size or shape, in a pipe or at the bottom or side wall of a container through which fluid is discharged. ✓ (1)

2. Orifices may be classified based on four factors:
   - Size ✓
   - Shape ✓
   - Sharpness ✓
   - Discharge conditions ✓ (4)

3. The three flow coefficients for an orifice are:
   - **Coefficient of velocity (C_v)** – It is defined as the ratio between the actual velocity of the jet and the theoretical velocity ✓

     $C_v = \frac{v_a}{v_t}$ ✓
   - **Coefficient of contraction (C_c)** – It is defined as the ratio between the actual area of the jet at the vena contracta and the theoretical area of the orifice. ✓

     $C_c = \frac{A_a}{A_t}$ ✓
   - **Coefficient of delivery (C_d)** – It is defined as the ratio between the actual discharge and the theoretical discharge. ✓

     $C_d = \frac{Q_a}{Q_t}$ ✓

     $C_d = C_c \times C_v$ (6)

4.1 $A_t = \pi \times 0{,}0{,}025^2 = 1{,}963 \times 10^{-3}\ m^2$ ✓

$A_a = C_c \times A_t = 0{,}65 \times 1{,}963 \times 10^{-3} = 1{,}276 \times 10^{-3}\ m^2$ ✓

$d = \sqrt{\frac{4A_a}{\pi}} = \sqrt{\frac{4 \times 1{,}276 \times 10^{-3}}{\pi}} = 40{,}311\ mm$ ✓ (3)

4.2 $v_t = \sqrt{2gh} = \sqrt{2 \times 9{,}81 \times 5} = 9{,}905\ m/s$ ✓

$v_a = C_v \times v_t = 0{,}95 \times 9{,}905 = 9{,}409\ m/s$ ✓ (2)

4.3 $Q_a = A_a \times v_a = 1{,}276 \times 10^{-3} \times 9{,}409 = 12{,}009 \times 10^{-3}\ m^3/s$ ✓

$\dot{m} = \rho Q_a = 10^3 \times 12{,}009 \times 10^{-3} = 12{,}009\ kg/s$ ✓ (2)

4.4 $t = \frac{V}{Q} = \frac{5}{12{,}009 \times 10^{-3}} = 416{,}36\ sec$ ✓ (1)

5.1 $A_a = \pi \times 0{,}016^2 = 804{,}248 \times 10^{-6}\ m^2$ ✓

$A_t = \pi \times 0{,}0{,}02^2 = 1{,}256 \times 10^{-3}\ m^2$ ✓

$C_c = \frac{A_a}{A_t} = \frac{804{,}248 \times 10^{-6}}{1{,}256 \times 10^{-3}} = 0{,}64$ ✓ (3)

5.2 $C_v = \frac{C_d}{C_c} = \frac{0{,}6}{0{,}64} = 0{,}938$ ✓ (1)

5.3 $v_t = \sqrt{2gh} = \sqrt{2 \times 9{,}81 \times 3} = 7{,}672\ m/s$ ✓

$v_a = C_v \times v_t = 0{,}938 \times 7{,}672 = 7{,}193\ m/s$ ✓

$\dot{m} = \rho A_a \times v_a = 800 \times 804{,}248 \times 10^{-6} \times 7{,}193 = 4{,}628\ kg/s$ ✓ (3)

5.4 $\frac{p_A}{\rho g} + \frac{v_A^2}{2g} + z_A = \frac{p_B}{\rho g} + \frac{v_B^2}{2g} + z_B$

$\frac{30 \times 10^3}{800 \times 9{,}81} + 0 + 3 = 0 + \frac{v_t^2}{2 \times 9{,}81} + 0$ ✓

$v_t = 11{,}57\ m/s$ ✓

$v_a = C_v \times v_t = 0{,}938 \times 11{,}57 = 10{,}847\ m/s$ ✓

$\dot{m} = \rho A_a v_a = 800 \times 804{,}248 \times 10^{-6} \times 10{,}847 = 6{,}979\ kg/s$ ✓ (4)

6.1 $A_a = \pi \times 0{,}0225^2 = 1{,}59 \times 10^{-3}\ m^2$ ✓

$A_t = \pi \times 0{,}0{,}025^2 = 1{,}963 \times 10^{-3}\ m^2$ ✓

$C_c = \frac{A_a}{A_t} = \frac{1{,}59 \times 10^{-3}}{1{,}963 \times 10^{-3}} = 0{,}81$ ✓ (3)

6.2 $h_L = h\left(1 - C_v^2\right)$

$1{,}5 = 5\left(1 - C_v^2\right)$ ✓

$C_v = 0{,}837$ ✓ (2)

6.3 $C_d = C_c \times C_v = 0{,}81 \times 0{,}837 = 0{,}678$ ✓ (1)

7.1 $\frac{p_A}{\rho g} + \frac{v_A^2}{2g} + z_A = \frac{p_B}{\rho g} + \frac{v_B^2}{2g} + z_B$

$0 + 0 + 2 = \frac{-35 \times 10^3}{10^3 \times 9{,}81} + \frac{v_t^2}{2 \times 9{,}81} + 0$ ✓

$v_t = 10{,}452$ m/s ✓

$v_a = C_v \times v_t = 0{,}8 \times 10{,}452 = 8{,}361$ m/s ✓ (3)

7.2 $\dot{m} = \rho A_a v_a = 10^3 \times 400 \times 10^{-6} \times 8{,}361 = 3{,}345$ kg/s ✓ (1)

Total: 40 marks

MODULE

10 Conservation of fluid momentum

By the end of this module, students should be able to:

- calculate the forces exerted by a jet on a flat surface (stationary and moving plate); and
- calculate the following:
  - horizontal force
  - vertical force components
  - resultant forces
  - angles.

Introduction

Fluid momentum is a concept in fluid mechanics that describes the motion of a fluid and its resistance to changes in motion. The design of many hydraulic structures, such as tapered pipes and bends, as well as blades in pumps and turbines, depends upon the forces that a fluid flow exerts on them. In this section, we will obtain these forces by using a linear momentum analysis, which is based on Newton's second law of motion. For application of this equation, it is important to calculate the change in the momentum that takes place.

Exercise 10.1

SB page 176

1.1 $\dot{m} = \rho Av = 10^3 \times \pi \times 0{,}035^2 \times 9 = 34{,}636$ kg/s

1.2 $F_d = \dot{m}v = 34{,}636 \times 9 = 311{,}725$ N

1.3 Take moments about the hinge:

$F_r \times y_2 = F_d \times y_1 F_r \times 0{,}5 = 282{,}743 \times 0{,}25 F_r = 155{,}862$ N

2.1 $\dot{m} = \rho Av = 10^3 \times \pi \times 0{,}02^2 \times 6 = 7{,}54$ kg/s

2.2 $F_d = \dot{m}v = 7{,}54 \times 6 = 45{,}239$ N

2.3 Take moments about the hinge:

$W \times x = F_d \times y5 \times 9{,}81 \times 0{,}25\sin\theta = 45{,}239 \times 0{,}25\theta = 67{,}265°$

3.1 $\dot{m} = \rho A v = 10^3 \times \pi \times 0{,}03^2 \times 7 = 19{,}792$ kg/s

3.2 $F_n = \dot{m} v \sin\theta = 19{,}792 \times 7\sin 60 = 119{,}983$ N

3.3 $F_x = F_n \sin\theta = 119{,}983 \times \sin 60 = 103{,}908$ N

3.4 $F_y = F_n \cos\theta = 96{,}264 \times \cos 60 = 59{,}991$ N

4.1 $\dot{m} = \rho A v = 10^3 \times \pi \times 0{,}019^2 \times 8 = 9{,}073$ kg/s

4.2 $F_x = \dot{m}(v + v\cos\theta) = 9{,}073(8 + 8\cos 20) = 140{,}789$ N

5.1 $F_x = \dot{m} v(1 + \cos\theta) = 15 \times 14(1 + \cos 45) = 358{,}492$ N

5.2 $F_y = \dot{m} v \sin\theta = 15 \times 14\sin 45 = 148{,}492$ N

5.3 $F_R = \sqrt{F_x^2 + F_y^2} = \sqrt{358{,}492^2 + 148{,}492^2} = 388{,}029$ N

$\beta = \tan^{-1}\left(\frac{F_y}{F_x}\right) = \tan^{-1}\left(\frac{148{,}492}{358{,}492}\right) = 22{,}5°$ (from horizontal downwards)

Exercise 10.2

SB page 181

1.1 $\dot{m}_e = \rho A(v - u) = 10^3 \times \pi \times 0{,}01^2(10 - 3) = 2{,}199$ kg/s

1.2 $F_d = \dot{m}_e \Delta v = \dot{m}_e(v - u) = 2{,}199(10 - 3) = 15{,}394$ N

1.3 $P_o = F_d \times u = 15{,}394 \times 3 = 46{,}181$ W

1.4 $\dot{m} = \rho A v = 10^3 \times \pi \times 0{,}01^2 \times 10 = 28{,}274$ kg/s

$P_i = \frac{\dot{m} v^2}{2} = \frac{3{,}142 \times 10^2}{2} = 157{,}08$ W

1.5 $\eta = \frac{P_o}{P_i} = \frac{46{,}181}{157{,}08} = 0{,}294 = 29{,}4\%$

2.1 $\dot{m}_e = \rho A(v - u) = 10^3 \times \pi \times 0{,}015^2(6 - 2) = 2{,}827$ kg/s

2.2 $F_n = \dot{m}_e(v - u)\sin\theta = 2{,}827(6 - 2)\sin 60 = 9{,}795$ N

2.3 $F_x = F_n \sin\theta = 9{,}785 \times \sin 60 = 8{,}482$ N

2.4 $P_o = F_x \times u = 8{,}482 \times 2 = 16{,}965$ W

2.5 $\dot{m} = \rho A v = 10^3 \times \pi \times 0{,}015^2 \times 6 = 4{,}241$ kg/s

$$P_i = \frac{\dot{m}v^2}{2} = \frac{4{,}241 \times 6^2}{2} = 76{,}341 \text{ W}$$

$$\eta = \frac{P_o}{P_i} = \frac{16{,}965}{76{,}341} = 0{,}222 = 22{,}222\%$$

3.1 $\dot{m}_e = \rho A(v - u) = 10^3 \times \pi \times 0{,}02^2(12 - 5) = 8{,}796 \text{ kg/s}$

3.2 $F_x = \dot{m}_e(v - u)(1 + \cos\theta) = 8{,}796 \times (12 - 5)(1 + \cos 70) = 82{,}635 \text{ N}$

3.3 $P_o = F_x \times u = 82{,}635 \times 5 = 413{,}176 \text{ W}$

3.4 $\dot{m} = \rho A v = 10^3 \times \pi \times 0{,}02^2 \times 12 = 15{,}08 \text{ kg/s}$

$$P_i = \frac{\dot{m}v^2}{2} = \frac{15{,}08 \times 12^2}{2} = 1\,085{,}734 \text{ W}$$

$$\eta = \frac{P_o}{P_i} = \frac{413{,}176}{1\,085{,}734} = 0{,}381 = 38{,}055\%$$

Exercise 10.3

SB page 187

1.1 $A_1 = \pi \times 0{,}15^2 = 70{,}686 \times 10^{-3} \text{ m}^2$

$\dot{m} = \rho A_1 v_1 = 10^3 \times 70{,}686 \times 10^{-3} \times 4 = 282{,}743 \text{ kg/s}$

1.2 $A_2 = \pi \times 0{,}075^2 = 17{,}671 \times 10^{-3} \text{ m}^2$

$$v_2 = \frac{A_1}{A_2}v_1 = \frac{70{,}686 \times 10^{-3}}{17{,}671 \times 10^{-3}} \times 4 = 16 \text{ m/s}$$

1.3 $\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2$

$$\frac{180 \times 10^3}{10^3 \times 9{,}81} + \frac{4^2}{2 \times 9{,}81} + 0 = \frac{p_2}{10^3 \times 9{,}81} + \frac{16^2}{2 \times 9{,}81} + 0$$

$p_2 = 60 \text{ kPa}$

1.4 $F_1 = p_1 A_1 + \dot{m}v_1 = 180 \times 10^3 \times 70{,}686 \times 10^{-3} + 282{,}743 \times 4 = 13{,}854 \text{ kN}$

1.5 $F_2 = p_2 A_2 + \dot{m}v_2 = 60 \times 10^3 \times 17{,}671 \times 10^{-3} + 282{,}743 \times 16 = 5{,}584 \text{ kN}$

1.6 $F_R = F_1 - F_2 = 13{,}854 - 5{,}584 = 8{,}27 \text{ kN}$

2.1 $\dot{m} = \rho Q = 850 \times 0{,}85 = 722{,}5 \text{ kg/s}$

2.2 $A_1 = \pi \times 0{,}3^2 = 282{,}743 \times 10^{-3} \text{ m}^2$

$$v_1 = \frac{Q}{A_1} = \frac{0{,}85}{282{,}743 \times 10^{-3}} = 3{,}006 \text{ m/s}$$

2.3 $A_2 = \pi \times 0{,}1^2 = 31{,}416 \times 10^{-3}\ m^2$

$v_1 = \frac{Q}{A_1} = \frac{0{,}85}{31{,}416 \times 10^{-3}} = 27{,}056\ m/s$

$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2$

$\frac{400 \times 10^3}{850 \times 9{,}81} + \frac{3{,}006^2}{2 \times 9{,}81} + 0 = \frac{p_2}{850 \times 9{,}81} + \frac{27{,}056^2}{2 \times 9{,}81} + 0$

$p_2 = 92{,}722\ kPa$

2.4 $F_1 = p_1 A_1 + \dot{m} v_1 = 400 \times 282{,}743 + 722{,}5 \times 3{,}006 = 115{,}269\ kN$

2.5 $F_2 = p_2 A_2 + \dot{m} v_2 = 92{,}722 \times 31{,}416 + 722{,}5 \times 27{,}056 = 22{,}461\ kN$

2.6 $F_R = \sqrt{F_1^2 + F_2^2 - 2 F_1 F_2 \cos\theta}$

$= \sqrt{115{,}269^2 + 22{,}461^2 - 2 \times 115{,}269 \times 22{,}461 \cos 60}$

$F_R = 105{,}842\ kN$

$\frac{\sin\alpha}{22{,}461} = \frac{\sin 60}{105{,}842}$

$\alpha = 10{,}59°$

Summative assessment: Module 10

SB page 190

1.1 $A = \pi \times 0{,}04^2 = 5{,}027 \times 10^{-3}\ m^2$ ✓

$\dot{m} = \rho A v = 10^3 \times 5{,}027 \times 10^{-3} \times 10 = 50{,}265\ kg/s$ ✓ (2)

1.2 $F_d = \dot{m} v = 50{,}265 \times 10 = 502{,}655\ N$ ✓ (1)

1.3 Take moments about the hinge:

$F_r \times y_2 = F_d \times y_1$

$F_r \times 0{,}4 = 502{,}655 \times 0{,}5$ ✓

$F_r = 628{,}319\ N$ ✓ (2)

1.4 Take moments about the hinge:

$F_d \times y = W \times x$

$F_d \times 0{,}5 = 40 \times 9{,}81 \times 0{,}4 \sin 40$ ✓

$F_d = 201{,}784\ N$ ✓

$v = \frac{F_d}{\dot{m}} = \frac{201{,}784}{50{,}265} = 4{,}015\ m/s$ ✓ (3)

2.1 $\dot{m} = \rho A v = 10^3 \times \pi \times 0{,}03^2 \times 7 = 19{,}792\ kg/s$ ✓ (1)

2.2 $F_n = \dot{m} v \sin\theta = 19{,}792 \times 7 \sin 30 = 69{,}272\ N$ ✓ (1)

2.3 $F_x = F_n \sin\theta = 69{,}272 \times \sin 30 = 34{,}636 \text{ N}$ ✓ (1)

2.4 $F_y = F_n \cos\theta = 69{,}272 \times \cos 30 = 59{,}991 \text{ N}$ ✓ (1)

3.1 $\dot{m} = \rho A v = 10^3 \times \pi \times 0{,}018^2 \times 9 = 9{,}161 \text{ kg/s}$ ✓ (1)

3.2 $F_x = \dot{m}(v + v\cos\theta) = 9{,}161(9 + 9\cos 45) = 140{,}747 \text{ N}$ ✓ (1)

4.1 $F_x = \dot{m}v(1 + \cos\theta) = 18 \times 15(1 + \cos 55) = 424{,}866 \text{ N}$ ✓ (1)

4.2 $F_y = \dot{m}v\sin\theta = 18 \times 15\sin 55 = 221{,}171 \text{ N}$ ✓ (1)

4.3 $F_R = \sqrt{F_x^2 + F_y^2} = \sqrt{424{,}866^2 + 221{,}171^2} = 478{,}986 \text{ N}$ ✓

$\beta = \tan^{-1}\left(\frac{F_y}{F_x}\right) = \tan^{-1}\left(\frac{221{,}171}{424{,}866}\right) = 27{,}5°$ ✓(from horizontal downwards)

(2)

5.1 $\dot{m}_e = \rho A(v - u) = 10^3 \times \pi \times 0{,}04^2(8 - 2) = 30{,}159 \text{ kg/s}$ ✓ (1)

5.2 $F_n = \dot{m}_e(v - u)\sin\theta = 30{,}159(8 - 2)\sin 70 = 170{,}043 \text{ N}$ ✓ (1)

5.3 $F_x = F_n \sin\theta = 170{,}043 \times \sin 70 = 159{,}788 \text{ N}$ ✓ (1)

5.4 $P_o = F_x \times u = 159{,}788 \times 2 = 319{,}576 \text{ W}$ ✓ (1)

5.5 $\dot{m} = \rho A v = 10^3 \times \pi \times 0{,}04^2 \times 8 = 40{,}212 \text{ kg/s}$ ✓

$P_i = \frac{\dot{m}v^2}{2} = \frac{40{,}212 \times 8^2}{2} = 1\,286{,}796 \text{ W}$ ✓

$\eta = \frac{P_o}{P_i} = \frac{319{,}576}{1\,286{,}796} = 0{,}248 = 24{,}835\%$ ✓ (3)

6.1 $\dot{m}_e = \rho A(v - u) = 10^3 \times \pi \times 0{,}025^2(8 - 3) = 9{,}817 \text{ kg/s}$ ✓ (1)

6.2 $F_x = \dot{m}_e(v - u)(1 + \cos\theta) = 9{,}817 \times (8 - 3)(1 + \cos 70) = 65{,}876 \text{ N}$ ✓ (1)

6.3 $P_o = F_x \times u = 65{,}876 \times 3 = 197{,}629 \text{ W}$ ✓ (1)

6.4 $\dot{m} = \rho A v = 10^3 \times \pi \times 0{,}025^2 \times 8 = 15{,}708 \text{ kg/s}$ ✓

$P_i = \frac{\dot{m}v^2}{2} = \frac{15{,}708 \times 8^2}{2} = 502{,}655 \text{ W}$ ✓

$\eta = \frac{P_o}{P_i} = \frac{197{,}629}{502{,}655} = 0{,}393 = 39{,}317\%$ ✓ (3)

7.1 $\dot{m} = \rho Q = 900 \times 0{,}9 = 810$ kg/s ✓ (1)

7.2 $A_1 = \pi \times 0{,}3^2 = 282{,}743 \times 10^{-3}$ m² ✓

$v_1 = \frac{Q}{A_1} = \frac{0{,}9}{282{,}743 \times 10^{-3}} = 3{,}183$ m/s ✓

$A_2 = \pi \times 0{,}2^2 = 125{,}667 \times 10^{-3}$ m² ✓

$v_1 = \frac{Q}{A_1} = \frac{0{,}9}{125{,}667 \times 10^{-3}} = 7{,}162$ m/s ✓

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2$$

$$\frac{p_1}{900 \times 9{,}81} + \frac{3{,}183^2}{2 \times 9{,}81} + 0 = \frac{100 \times 10^3}{900 \times 9{,}81} + \frac{7{,}162^2}{2 \times 9{,}81} + 0$$ ✓

$p_2 = 118{,}523$ kPa ✓ (6)

7.3 $F_1 = p_1 A_1 + \dot{m} v_1 = 118{,}523 \times 282{,}743 + 810 \times 3{,}183$ ✓ $= 36{,}09$ kN ✓ (2)

7.4 $F_2 = p_2 A_2 + \dot{m} v_2 = 100 \times 125{,}667 + 810 \times 7{,}162$ ✓ $= 18{,}368$ kN ✓ (2)

7.5 $F_R = \sqrt{F_1^2 + F_2^2 - 2 F_1 F_2 \cos\theta}$

$= \sqrt{36{,}09^2 + 18{,}368^2 - 2 \times 36{,}09 \times 18{,}368 \cos 120}$ ✓

$F_R = 47{,}987$ kN ✓

$$\frac{\sin\alpha}{18{,}368} = \frac{\sin 120}{47{,}987}$$

$\alpha = 19{,}359°$ ✓ (3)

Total: 45 marks

MODULE

11 Reciprocating pumps

By the end of this module, students should be able to:

- draw a line diagram for a reciprocating pump, indicating the following components:
  - suction head
  - delivery head
  - static head;
- explain the following concepts:
  - velocity head
  - friction head
  - separation head
  - manometric head; and
- perform calculations on the flow rate for single- and double-acting reciprocating pumps (including slip and output and input power).

Introduction

Pumps are used to increase the energy level of water in order for it to be raised to a higher level. A reciprocating pump is also known as a positive displacement pump because it discharges a definite quantity of liquid. The delivery of a reciprocating pump is determined by the physical dimensions of the piston diameter and stroke length as well as the speed of the pump. The discharge of the pump is independent of the head against which the pump is operating. It is often used where a small quantity of liquid is to be handled and where delivery pressure is high.

A reciprocating pump is a device that converts mechanical energy into hydraulic energy by drawing the liquid into a cylinder. A piston or plunger reciprocates which then exerts pressure on the liquid and increases its hydraulic energy. The piston or plunger slides inside a cylinder. The pump utilises a crankshaft-connecting rod mechanism identical to internal combustion engines. The crankshaft-connecting rod mechanism converts the rotary movement of the crankshaft to a reciprocating linear motion of plungers or pistons. A suction pipe allows fluid to be drawn into the chamber at low pressure, while a delivery pipe allows fluid to be discharged at high pressure. Inlet and outlet valves check the flow of fluid into and out of the cylinder.

Exercise 11.1

SB page 200

1.1 $A = \pi \times 0{,}1^2 = 31{,}416 \times 10^{-3}\ \text{m}^2$

$$\ell = \frac{Q_a}{C_d AnEN} = \frac{120 \times 10^{-3}}{0{,}97 \times 31{,}416 \times 10^{-3} \times 2 \times 2 \times 2{,}5} = 393{,}785\ \text{mm}$$

1.2 $p = \rho g H_s = 10^3 \times 9{,}81 \times 18 = 176{,}58\ \text{kPa}$

1.3 $P_o = pQ_a = 176{,}58 \times 10^3 \times 120 \times 10^{-3} = 21{,}19\ \text{kW}$

1.4 $P_i = \frac{P_o}{\eta} = \frac{21{,}19 \times 10^3}{0{,}85} = 24{,}929\ \text{kW}$

2.1 $A = \frac{Q_a}{C_d \ell nEN} = \frac{500 \times 10^{-3}}{0{,}98 \times 500 \times 10^{-3} \times 3 \times 1 \times 3} = 113{,}379 \times 10^{-3}\ \text{m}^2$

$$d = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{4 \times 113{,}379 \times 10^{-3}}{\pi}} = 379{,}945\ \text{mm}$$

2.2 $p = \rho g H_s = 10^3 \times 9{,}81 \times 10 = 98{,}1\ \text{kPa}$

2.3 $P_o = pQ_a = 98{,}1 \times 10^3 \times 500 \times 10^{-3} = 49{,}05\ \text{kW}$

2.4 $\eta = \frac{P_o}{P_i} = \frac{49{,}05 \times 10^3}{65 \times 10^3} = 0{,}755 = 75{,}462\%$

Summative assessment: Module 11

SB page 202

1.1 $A = \pi \times 0{,}1^2 = 31{,}416 \times 10^{-3}\ \text{m}^2$ ✓

$Q_t = \ell AnEN = 0{,}4 \times 31{,}416 \times 10^{-3} \times 2 \times 2 \times 5 = 251{,}327 \times 10^{-3}\ \text{m}^3/\text{s}$ ✓

$C_d = \frac{Q_a}{Q_t} = \frac{245 \times 10^{-3}}{251{,}327 \times 10^{-3}} = 0{,}975$ ✓

$\%\text{slip} = 1 - C_d = 1 - 0{,}975 = 0{,}025 = 2{,}518\%$ ✓ (4)

1.2 $H_s = \frac{p}{\rho g} = \frac{150 \times 10^3}{10^3 \times 9{,}81} = 15{,}291\ \text{m}$ ✓ (1)

1.3 $P_o = pQ_a = 150 \times 10^3 \times 245 \times 10^{-3} = 36{,}75\ \text{kW}$ ✓ (1)

1.4 $P_i = \frac{P_o}{\eta} = \frac{36{,}75 \times 10^3}{0{,}8} = 45{,}938\ \text{kW}$ ✓ (1)

2.1 $A = \pi \times 0{,}05^2 = 7{,}854 \times 10^{-3}\ m^2$ ✓

$Q_a = C_d \ell AnEN = 0{,}97 \times 0{,}3 \times 7{,}854 \times 10^{-3} \times 1{,}5 = 3{,}428 \times 10^{-3}\ m^3/s$ ✓

$\dot{m} = \rho Q = 10^3 \times 3{,}428 \times 10^{-3} = 3{,}428\ kg/s$ ✓ (3)

2.2 $H_s = \frac{p}{\rho g} = \frac{196{,}2 \times 10^3}{10^3 \times 9{,}81} = 20\ m$ ✓

$h_d = H_s - h_s = 20 - 3 = 17\ m$ ✓ (2)

2.3 $p = \rho g h_d = 10^3 \times 9{,}81 \times 17 = 166{,}77\ kPa$ ✓

$F = pA = 166{,}77 \times 10^3 \times 7{,}854 \times 10^{-3} = 1{,}31\ kN$ ✓ (2)

2.4 $P_i = \frac{pQ}{\eta} = \frac{196{,}2 \times 10^3 \times 3{,}428 \times 10^{-3}}{0{,}85} = 791{,}324\ W$ ✓ (1)

Total: 15 marks

Glossary

A

Actuating – to start or put something into action

B

Ballast –a heavy and dense object used to improve stability and control

Barometer – an instrument that measures atmospheric pressure

Bell-mouthed orifice – an orifice with a rounded taper which guides flow

Boiling point – the temperature at which a liquid turns into vapour

Bourdon gauge – a type of pressure gauge made of a tube which is bent into an arc or coil

Bulk modulus – the ratio between the change in pressure and volumetric strain

Buoyancy – the tendency of an immersed body to be lifted up in a fluid due to an upward force opposite to the action of gravity

C

Cavitation – the formation of bubbles in a liquid due to pressure

Compressibility – the ability to change volume under pressure

Concentric – circles or rings with a common centre

D

Deform – change shape or become distorted

Density – an object or substance's mass per unit volume

Descent weights – ballast tanks filled with water to increase a submarine's weight to allow for descending

Differential manometer – an instrument used to measure the difference of pressures between two points in a pipe or different pipes

Dynamic – change or activity in a system or process as brought about by a force

E

Equilibrium – a state of balanced or equal forces

Exert – to apply

F

Fluid – a substance in the liquid or gas phase

G

Gauge – a measuring instrument that displays or indicates results

H

Hydraulic – the use of fluids under pressure to generate mechanical power

Hydrostatics – a branch of physics that deals with the characteristics of fluids at rest

I

Immersed body – an object completely surrounded by or in contact with fluid

Immiscible – liquids that cannot be mixed together or combined into a homogenous mixture

K

Kinematic – relating to motion

L

Laminar flow – a type of flow in which the fluid particles move along well-defined paths or streamlines which are straight and parallel

M

Manometer – an instrument that measures the pressure acting on fluid in a confined space

Metacentre – the intersection point of the vertical through the centre of buoyancy of a floating body and the vertical through the new centre of buoyancy after the body is displaced

Metacentric height – the measurement of initial static stability of a floating body

O

Orifice – an opening, of any size or shape, in a pipe or at the bottom or side wall of a container through which fluid is discharged

P

Pathline – the position of a particle over a period of time

Piezo tube – actuators that contract when a voltage is applied between the inner and outer electrodes

Piezometer – an instrument that measures the pressure or compressibility of a substance

R

Radial clearance – the total distance a ring can be moved in relation to another ring

Reciprocal – the inverse of a number when multiplied

Reservoir – a large body of water or water supply

Reynolds number – a dimensionless quantity that helps predict fluid flow patterns in different situations by measuring the ratio between different forces

S

Sluice gate – a sliding gate used to control or measure water flow

Specific gravity – another term for relative density

Specific weight – the weight per unit volume of an object or substance

Stability – the ability of a body in a fluid to return to its original position after being tilted about a horizontal axis

Stagnation pressure – the pressure at a point when the fluid comes to rest

Streamline – the curve that joins the position of successive particles over a period of time

Streamtube – the combination of a number of streamlines flowing together to form a tube

Surface tension – the tension of the surface film of a liquid caused by the attraction of particles in the surface layer by the bulk of the liquid

Syntactic foam – a composite material used to improve buoyancy in submarines and similar vessels

T

Turbulent flow – the type of flow in which the fluid particles move in a zig-zag or irregular way

V

Vacuum – a space that is empty or doesn't contain matter

Variable ballast spheres – ballast spheres used to control

Vena contracta – the point in the fluid stream where the cross-sectional area of the stream is the least and fluid velocity is at its maximum

Venturi meter – type of differential pressure flow meter that generates flow measurement by measuring the pressure difference at two different locations in a pipe

Viscosity – the measurement of the fluid's resistance to flow

Viscous resistance – the effect of surface friction when particles move through a liquid