higher education & training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

NASIENRIGLYN

NASIONALE SERTIFIKAAT

WISKUNDE N3

24 November 2023

Hierdie nasienriglyn bestaan uit 11 bladsye.

NASIENSLEUTEL	
SIMBOOL	**VERKLARING VAN SIMBOOL**
A	Akkuraatheid
P	Metode
CA	Konsekwent akkuraat (opvolg)
B/D	Uiteensetting
S	Stelling
R	Rede
S/R	Stelling en rede
F	Kies die korrekte formule
SF	Vervanging in die korrekte formule

Kopiereg voorbehou

VRAAG 1

	OPLOSSING	VERKLARING	
1.1	$(p+qn)(p-qn)+2qn-1$ $=p^2-q^2n^2+2qn-1$ ✓ $=p^2-(q^2n^2-2qn+1)$ ✓ $=p^2-[(qn-1)(qn-1)]$ $=[p-(qn-1)][p+(qn-1)]$ ✓ ✓	1 Vereenvoudiging 1 Groepering 1 punt vir elke faktor	(4)
1.2	$f(x)=2x^3+4ax^2-3a^2x-2$ $R=f(a)=2(a)^3+4a(a)^2-3a^2(a)-2$ ✓ ✓ ✓ $=2a^3+4a^3-3a^3-2$ $\therefore \quad 22=3a^3-2$ $\therefore \quad 3a^3=24$ $a^3=8$ $\therefore a=2$ ✓	1 vir R=f(a) 1 SF 1 S 1 A	(4)
1.3	$\frac{a^2-a-6}{a^2+a-20}\times\frac{2a-8}{2a-6}\div\frac{a^2-3a-10}{a^2-25}$ $=\frac{a^2-a-6}{a^2+a-20}\times\frac{2a-8}{2a-6}\times\frac{a^2-25}{a^2-3a-10}$ ✓ ✓ $=\frac{(a-3)(a+2)}{(a-4)(a+5)}\times\frac{2(a-4)}{2(a-3)}\times\frac{(a-5)(a+5)}{(a+2)(a-5)}$ √ √ √ √ √ √ $=1$ ✓	2 P ½ vir korrekte teller, ½ vir korrekte noemer 1 A	(6)

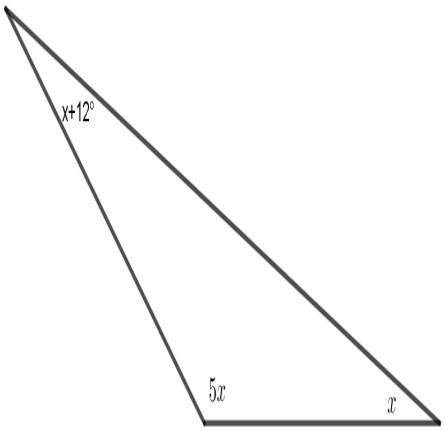
	OPLOSSING		VERKLARING	
1.4	$4x^2 - x - 1 = 0$ $\therefore \quad 4x^2 - x = 1$ $\therefore \quad x^2 - \frac{1}{4}x = \frac{1}{4}$ ✓ $\therefore x^2 - \frac{1}{4}x + \left(-\frac{1}{4\times 2}\right)^2 = \frac{1}{4} + \left(-\frac{1}{4\times 2}\right)^2$ $\therefore x^2 - \frac{1}{4}x + \left(-\frac{1}{8}\right)^2 = \frac{1}{4} + \frac{1}{64}$ ✓ $\therefore \quad \left(x - \frac{1}{8}\right)^2 = \frac{16+1}{64}$ $\therefore \quad \left(x - \frac{1}{8}\right)^2 = \frac{17}{64}$ $\therefore \quad x - \frac{1}{8} = \pm\frac{\sqrt{17}}{8}$ $\therefore \quad x = \frac{1}{8} \pm \frac{\sqrt{17}}{8}$ ✓ $\therefore \quad x = \frac{1+\sqrt{17}}{8}$ or $x = \frac{1-\sqrt{17}}{8}$ $\therefore \quad x = 0,640$ ✓ $x = -0,390$ ✓		1 Vereenvoudiging 1 voltooiing van die vierkant 1 Vereenvoudiging 1 punt vir elke waarde van x	(5)
				[19]

VRAAG 2

	OPLOSSING		VERKLARING	
2.1	$\frac{20^{2n+1}\times 100^{2-n}}{64^{\frac{n}{3}}}$ $= \frac{(5\times 2^2)^{2n+1}\times(5^2\times 2^2)^{2-n}}{(2^6)^{\frac{n}{3}}}$ ✓ $= \frac{5^{2n+1}\times 2^{4n+2}\times 5^{4-2n}\times 2^{4-2n}}{2^{2n}}$ $= 5^{2n+1+4-2n}\times 2^{4n+2+4-2n-2n}$ √ √ $= 5^3 \times 2^6$ √ $= 3125\times 64$ $= 200\ 000$ ✓	ALTERNATIEWE OPLOSSING $\frac{20^{2n+1}\times 100^{2-n}}{64^{\frac{n}{3}}}$ $= \frac{(2\times 10)^{2n+1}\times(10^2)^{2-n}}{(2^6)^{\frac{n}{3}}}$ ✓ $= \frac{2^{2n+1}.10^{2n+1}.10^{4-2n}}{2^{2n}}$ √ √ $= 2^{2n+1-2n}.10^{2n+1+4-2n}$ $= 2\times 10^5$ √ √ $= 200\ 000$ ✓	1 deling van die magte 2 Vereenvoudiging van magte 1 A	(4)

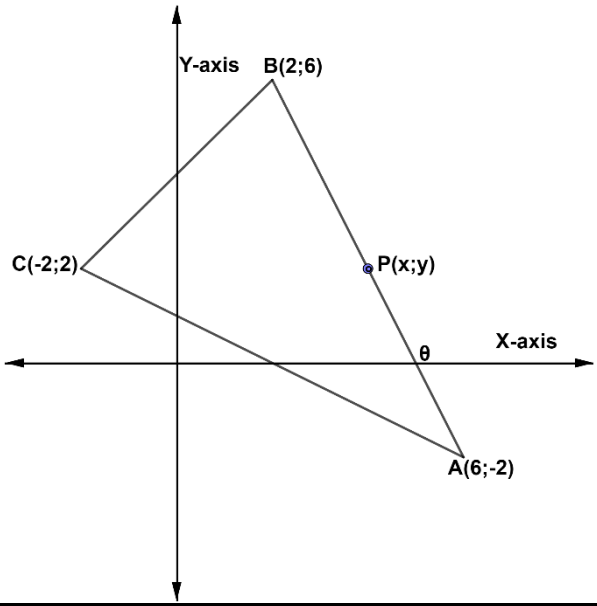
		OPLOSSING		**VERKLARING**	
2.2	2.2.1	$(\log x)^2 - 5 = \log(10x)$ $\therefore (\log x)^2 - 5 = \log 10 + \log x$ ✓ $\therefore (\log x)^2 - \log x - 1 - 5 = 0$ $\therefore (\log x)^2 - \log x - 6 = 0$ ✓ $\therefore (\log x - 3)(\log x + 2) = 0$ ✓ $\therefore \log x = 3$ or $\log x = -2$ $\therefore x = 10^3$ $x = 10^{-2}$ $\therefore x = 1000$ ✓ or $x = \frac{1}{100}$ ✓		1 Aanwending van die log-wet 1 Vereenvoudiging 1 Faktorisering 1 vir elke waarde van x	(5)
	2.2.2	$3(x^{\frac{1}{2}} + x^{-\frac{1}{2}}) = 10$ $3x^{\frac{1}{2}} + \frac{3}{x^{\frac{1}{2}}} = 10$ ✓ $3x + 3 = 10x^{\frac{1}{2}}$ $3x - 10x^{\frac{1}{2}} + 3 = 0$ ✓ $(3x^{\frac{1}{2}} - 1)(x^{\frac{1}{2}} - 3) = 0$ ✓ $(3x^{\frac{1}{2}} - 1) = 0$ or $(x^{\frac{1}{2}} - 3) = 0$ $x^{\frac{1}{2}} = \frac{1}{3}$ or $x^{\frac{1}{2}} = 3$ $\therefore x = \frac{1}{9}$ ✓ or $x = 9$ ✓		1 S 1 S 1 Faktorisering 1 vir x: 1 vir x	(5)
					[14]

VRAAG 3

	OPLOSSING		VERKLARING	
3.1	$BA^2 + DA = V$ $BA^2 + DA - V = 0$ ✓ $A = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $A = \frac{-D \pm \sqrt{D^2 - 4(B)(-V)}}{2B}$ ✓ $A = \frac{-D \pm \sqrt{D^2 + 4BV}}{2B}$ ✓	ALTERNATIEWE ANTWOORD $BA^2 + DA = V$ $\therefore A^2 + \frac{D}{B}A = \frac{V}{B}$ $\therefore A^2 + \frac{D}{B}A + \frac{D^2}{4B^2} = \frac{D^2}{4B^2} + \frac{V}{B}$ ✓ $\therefore \left(A + \frac{D}{2B}\right)^2 = \frac{D^2 + 4BV}{4B^2}$ $\therefore A + \frac{D}{2B} = \pm\sqrt{\frac{D^2 + 4BV}{4B^2}}$ ✓ $= \pm\frac{\sqrt{D^2 + 4BV}}{2B}$ $\therefore A = -\frac{D}{2B} \pm \frac{\sqrt{D^2 + 4BV}}{2B}$ $= \frac{-D \pm \sqrt{D^2 + 4BV}}{2B}$ ✓	1 S 1 SF 1 A	(3)
3.2	Laat die eerste hoek x wees $\therefore$ die tweede hoek sal $5x$ wees En die derde hoek sal $x + 12^\circ$ wees $x + 5x + x + 12^\circ = 180^\circ$ ✓ $\therefore 7x = 180^\circ - 12^\circ$ ✓ $\therefore 7x = 168^\circ$ $\therefore x = 24^\circ$ ✓ $\therefore 5x = 120^\circ$ ✓ $\therefore x + 12^\circ = 36^\circ$ ✓ Die hoeke is: 24°; 120° *en* 36°		1 S 1 vereenvoudiging 1 vir elke antwoord	(5)

	OPLOSSING		VERKLARING	
3.3	$\frac{x+1}{x-1}=\frac{-2}{x+2}\checkmark\frac{x+2}{1-x}$ ✓ $\therefore \frac{x+1}{x-1}=\frac{-2}{x+2}+\frac{x+2}{x-1}$ $\therefore (x+1)(x+2)=-2(x-1)+(x+2)(x+2)$ ✓ ✓ ✓ $\therefore x^2+3x+2=-2x+2+x^2+4x+4$ $\therefore \quad 3x \quad =-2x+4x+4$ $\therefore \quad x=4$ ✓		1 P uithaal -1 as algemene faktor 1 P vir vermenigvuldiging van elke breuk deur die KGN ('LCD') 1 vir antwoord	(5)
				[13]

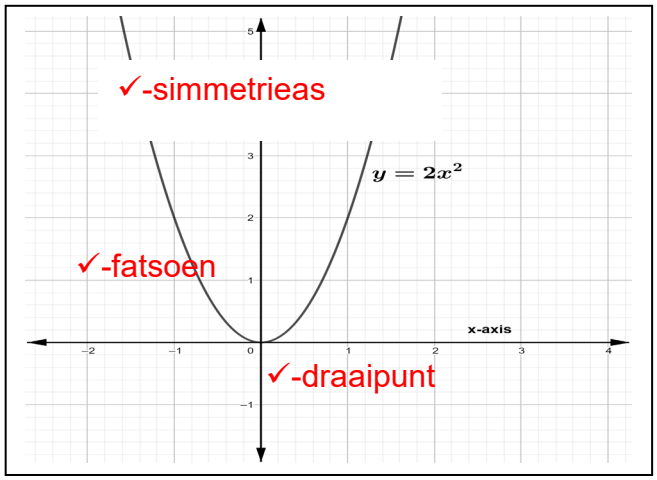
VRAAG 4

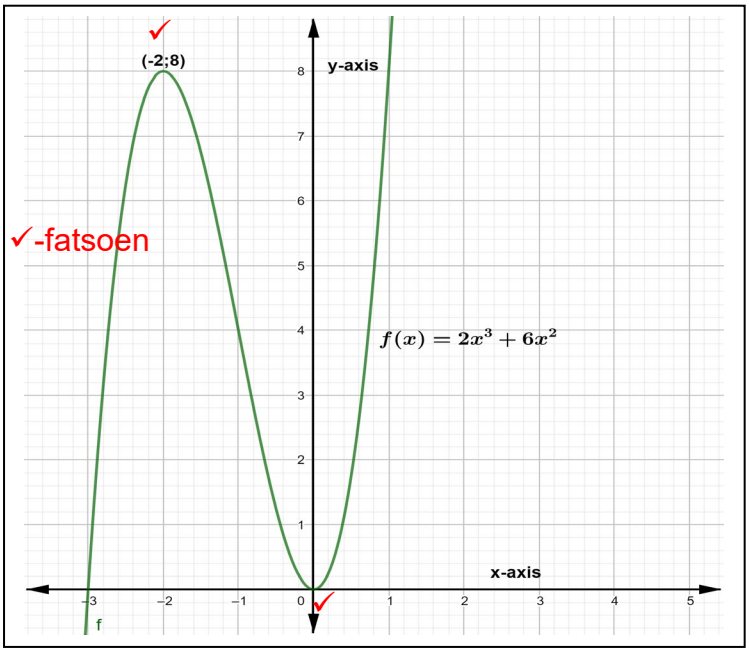
	OPLOSSING		VERKLARING	
				
4.1	$M_{AB}=\frac{y_2-y_1}{x_2-x_1}$ $=\frac{6+2}{2-6}$ ✓ $=\frac{8}{-4}$ $=-2$ ✓		1 SF 1 A	(2)

	OPLOSSING		VERKLARING	
4.2	$\theta = \tan^{-1}(-2)$ ✓ $\therefore \theta = 180^\circ - 63,435^\circ$ ✓ $\therefore \theta = 116,565^\circ$ ✓	ALTERNATIEWE ANTWOORD ✓ $\theta = \tan^{-1}(-2)$ $\therefore \theta = -63,435^\circ + 180^\circ$ ✓ $\therefore \theta = 116,565^\circ$ ✓	1 CA 1 SF 1 A	(3)
4.3	Midpoint P: $= \left(\frac{x_1+x_2}{2}; \frac{y_1+y_2}{2}\right)$ $= \left(\frac{6+2}{2}; \frac{-2+6}{2}\right)$ ✓ $= \left(\frac{8}{2}; \frac{4}{2}\right)$ $= (4;2)$ ✓		1 SF 1 A	(2)
4.4	Distance $AC = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ $= \sqrt{(-2-6)^2+(2+2)^2}$ ✓ $= \sqrt{64+16}$ $= \sqrt{80}$ $= \sqrt{16\times 5}$ √ $= 4\sqrt{5}$ √		1 SF ½ P vir antwoord ½ P vir eenvoudigste wortelvorm	(2)
4.5	$y - y_1 = m(x - x_1)$ $y - (-2) = 0(x - 6)$ $\therefore y = 2$ ✓ ✓		Indien die student y=2 geskryf het, ken vol punt vir die antwoord toe. Dit was nie nodig om aan te dui dat die gradiënt sero is nie.	(2)
4.6	$\therefore x = 2$ ✓		Indien die student x=2 geskryf het, ken vol punt vir die antwoord toe. Dit was nie nodig om aan te dui dat die gradiënt ongedefinieerd is nie. Alle x-koördinate is gelyk.	(1)
4.7	Snypunt $= (-4;7)$ ✓		1 P	(1)
4.8	$y - y_1 = \frac{y_2-y_1}{x_2-x_1}(x - x_1)$ $\therefore y - 0 = \frac{6-0}{2-0}(x - 0)$ ✓ $\therefore y = 3x$ ✓		1 SF 1 A	(2)

4.9	$M_{BC} = 1$ ✓ $M_{BC} \times M = -1$ $1 \times M = -1$ $M = -1$ ✓	$y - y_1 = m(x - x_1)$ $y - 6 = -1(x - 2)$ ✓ $y = -x + 2 + 6$ $y = -x + 8$ ✓	1 P vir gradiënt (m) = -1 1 SF 1 antwoord	(4)
				[19]

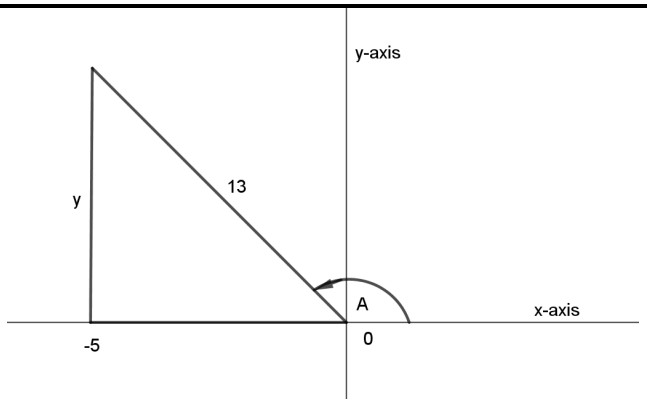
VRAAG 5

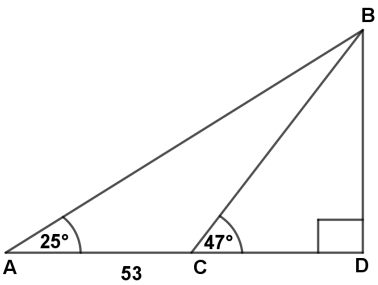
	OPLOSSING		VERKLARING	
5.1	$y = 2x^2$  		1 Draaipunt (0;0) 1 simmetrieas 1 fatsoen Die punt vir die fatsoen kan slegs toegeken word indien die grafiek 'n min TP het, in die eerste en tweede kwadrant geteken is en oor 'n draaipunt van (0;0) beskik.	(3)
5.2	$y = \frac{2}{x} - 2\sqrt{x} + \frac{1}{3}x^3$ $= 2x^{-1} - 2x^{\frac{1}{2}} + \frac{1}{3}x^3$ √ √ $\frac{dy}{dx} = -1 \times 2x^{-1-1} - \frac{1}{2} \times 2x^{\frac{1}{2}-1} + \frac{3x^{3-1}}{3}$ $= -2x^{-2} - x^{-\frac{1}{2}} + x^2$ ✓ ✓ ✓ $= \frac{-2}{x^2} - \frac{1}{\sqrt{x}} + x^2$ √ √		½ P elk vir die gebruik van die eksponentwette in stap 2 1 punt vir elke derivaat ½ vir magte in die korrekte vorm	(5)

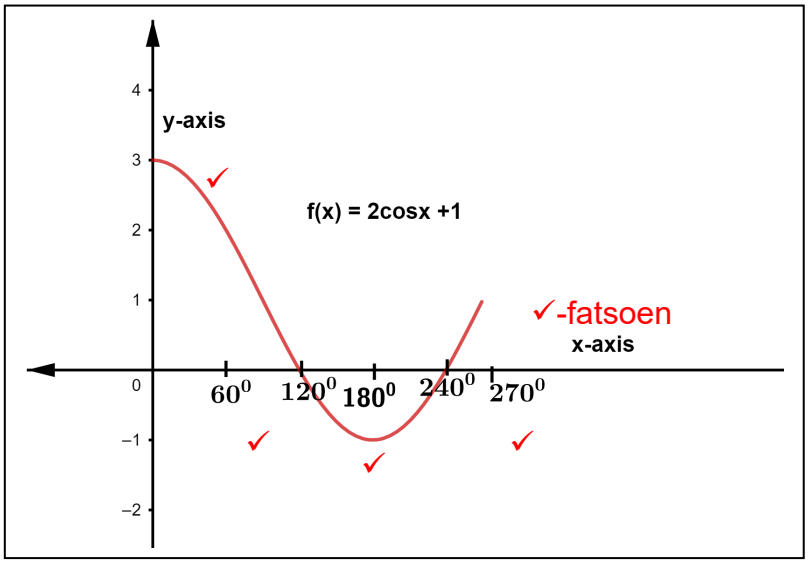
		OPLOSSING		VERKLARING	
5.3	5.3.1	$f(x) = 2x^3 + 6x^2$ $f(x) = 2x^3 + 6x^2$ $\therefore f'(x) = 6x^2 + 12x$ √ For TP's : $f'(x) = 0$ √ $\therefore\ 6x^2 + 12x = 0$ $\therefore 6x(x+2) = 0$ $\therefore x = 0$ ✓ *or* $x = -2$ ✓ If $x = 0$: $\therefore f(0) = 2(0)^3 + 6(0)^2 = 0$ ✓ If $x = -2$: $\therefore f(-2) = 2(-2)^3 + 6(-2)^2$ $= -16 + 24$ $= 8$ ✓ Die koördinate van die draaipunte is $(0;0)$ en $(-2;8)$		½ P vir derivaat ½ S (as dit gelyk is aan sero - vir gebruik van kennis oor min en maks draaipunte) 1 P elk vir die bepaling van x 1 P elk vir die bepaling van y	(5)
	5.3.2	$2x^3 + 6x^2 = 0$ $2x^2(x+3) = 0$ $x = 0$ *and* $x = -3$  		1 P vir elke draaipunt 1 P vir fatsoen 1 vir fatsoen indien al die ander inligting korrek was. Die maks DP is (-2;8) en die min DP is (0;0)	(3)
					[16]

VRAAG 6

	OPLOSSING	VERKLARING	
6.1	$\frac{\tan A}{1+\tan^2 A} = \sin A \cos A$ $LHS = \frac{\tan A}{1+\tan^2 A}$ $= \frac{\tan A}{\sec^2 A}$ ✓ $= \frac{\sin A}{\cos A} \times \frac{1}{\sec^2 A}$ ✓ $= \frac{\sin A}{\cos A} \times \cos^2 A$ ✓ $= \sin A \cos A$ ✓ $\therefore LHS = RHS$	1 P vierkant-identiteit 1 P kwosiënt-identiteite 1 P 1 R	(4)

	OPLOSSING		VERKLARING	
6.2	$y^2 + 25 = 169$ $\therefore y^2 = 144$ $\therefore y = 12$			
	6.2.1	$\therefore \tan A = \frac{12}{-5}$ ✓ ✓	1 vir bepaling van y en vervangings met 12 1 vir noemer	(2)
	6.2.2	✓ $\sec A = -\frac{13}{5}$ ✓	1 vir teller 1 vir noemer	(2)

6.3	*In* Δ ABC: $A\hat{C}B=133^o$ $A\hat{B}C= 22^o$ $\therefore \frac{BC}{\sin 25^o}=\frac{53}{\sin 22^o}$ ✓ $\therefore \quad BC=\frac{53\sin 25^o}{\sin 22^o}$ ✓ $\therefore \quad BC=59,793$ ✓ *In* Δ BCD: $\frac{BD}{BC}=\sin 47^o$ ✓ $\therefore BD = 59,793\sin 47^o$ ✓ $=43,730$ m ✓	 **ALTERNATIEWE ANTWOORD** *In* Δ ABC: $A\hat{C}B=133^o$ $A\hat{B}C= 22^o$ $\therefore \frac{AB}{\sin 133^o}=\frac{53}{\sin 22^o}$ ✓ $\therefore \quad AB=\frac{53\sin 133^o}{\sin 22^o}$ ✓ $\therefore \quad AB=103,473$ ✓ *In* Δ ABD: ✓ $\frac{BD}{AB}=\sin 25^o$ $\therefore BD = 103,473\sin 25^o$ ✓ $=43,730$ m ✓	1 S 1 SF 1 antwoord 1 S 1 SF 1 antwoord	(6)

	OPLOSSING		**VERKLARING**	
6.4			1 vir DP (;-1)180° 1 vir DP (0° ;3)/of y-afsnit x-afsnitte (120° ;0) x-afsnit (240° ; 0) 1 P fatsoen	(5)
				[19]

TOTAAL: 100

Kopiereg voorbehou