higher education & training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

MARKING GUIDELINE

NATIONAL CERTIFICATE

MATHEMATICS N3

18 November 2022

This marking guideline consists of 16 pages.

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INSTRUCTIONS TO MARKERS

✓=1 mark
√ = ½ mark

1. Only use a red pen.

2. If a candidate answers a question twice and does not delete either, mark the first attempt.

3. If a candidate answers a question, crosses it out and does not redo it, mark the deleted attempt.

MARKING LEGEND	
SYMBOL	**EXPLANATION OF SYMBOL**
A	Accuracy
M	Method
CA	Consistent accuracy (follow up)
B/D	Breakdown
S	Statement
R	Reason
S/R	Statement and reason
F	Choosing correct formula
SF	Substitution into correct formula

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QUESTION 1

		SOLUTION	EXPLANATION
1.1	1.1.1.	$-x^3-4y^2+x^2y+4xy$ $=\left(-x^3+x^2y\right)+\left(-4y^2+4xy\right)$ $=-x^2(x-y)+4y(x-y)$ $=(x-y)(4y-x^2)$	½ M for each correct group ½ correct CF ½ each factor
		Alternative solution $-x^3-4y^2+x^2y+4xy$ $=\left(-x^3+4xy\right)+\left(-4y^2+x^2y\right)$ $=-x\left(x^2-4y\right)+y\left(x^2-4y\right)$ $=\left(x^2-4y\right)(-x+y)$	½ M for each correct group ½ correct CF ½ each factor (3)
	1.1.2.	$b^2-4bd+4d^2-(2b+d)^2$ $=(b-2d)(b-2d)-(2b+d)^2$ $=(b-2d)^2-(2b+d)^2$ $=\left[(b-2d)-(2b+d)\right]\left[(b-2d)+(2b+d)\right]$ ✓ $=[b-2d-2b-d][b-2d+2b+d]$ ✓ $=(-b-3d)(3b-d)$	1 factorising trinomial 1 for factorising using difference of two squares 1 mark for simplifying ½ mark each simplified factor
		Alternative solution $b^2-4bd+4d^2-(2b+d)^2$ $=b^2-4bd+4d^2-\left(4b^2+4bd+d^2\right)$ $=b^2-4b^2-4bd-4bd+4d^2-d^2$ ✓ $=-3b^2-8bd+3d^2$ ✓ $=(-3b-d)(-b-3d)$	2 simplifying the binomial 1 simplifying the polynomial ½ each simplified factor (4)
1.2			1 M completing square

$x^2 - 4x + 2 = 0$

$\therefore x^2 - 4x = -2$

$\therefore x^2 - 4x + \left(\frac{-4}{2}\right)^2 = -2 + \left(\frac{-4}{2}\right)^2$ ✓

$\therefore (x-2)^2 = -2 + 4$

$\therefore x - 2 = \pm\sqrt{2}$ ✓

$\therefore \quad x = 2 \pm \sqrt{2}$

$\therefore \quad x = 3{,}414$ ✓ or $x = 0{,}586$ ✓

1 for simplifying

1 mark for each accurate answer

(4)

1.3 $h(x) = 3x^3 - 9x^2 + 12$

$R = h(-1) = 3(-1)^3 - 9(-1)^2 + 12$

$= -3 - 9 + 12$

$= 0$ ✓

$\therefore x + 1$ is a factor of $h(x)$

$$\begin{array}{r} 3x^2 - 12x + 12 \\ x+1 \overline{)\, 3x^3 - 9x^2 + 0x + 12} \\ \underline{3x^3 + 3x^2} \\ -12x^2 + 0x \\ \underline{-12x^2 - 12x} \\ 12x + 12 \\ \underline{12x + 12} \\ 0 \end{array}$$ ✓ ✓

$h(x) = (x+1)(3x^2 - 12x + 12)$

$\therefore h(x) = 3(x+1)(x-2)(x-2)$

$\therefore h(x) = 3(x+1)(x-2)^2$ ✓ ✓ ✓

Alternative solution 1

$h(x) = 3x^3 - 9x^2 + 12$

$$\begin{array}{r|rrrr} -1 & 3 & -9 & 0 & 12 \\ & & -3 & 12 & -12 \\ \hline & 3 & -12 & 12 & 0 \end{array}$$ ✓ ✓ ✓

$\therefore f(x) = (x+1)(3x^2 - 12x + 12)$ ✓ ✓

$\therefore f(x) = 3(x+1)(x-2)^2$ ✓ ✓

Alternative solution 2

$$\begin{array}{r} 3x^2 - 12x + 12 \\ x+1 \overline{)\, 3x^3 - 9x^2 + 0x + 12} \\ \underline{3x^3 + 3x^2} \\ -12x^2 + 0x \\ \underline{-12x^2 - 12x} \\ 12x + 12 \\ \underline{12x + 12} \\ 0 \end{array}$$ ✓ ✓ ✓

$h(x) = (x+1)(3x^2 - 12x + 12)$

$\therefore h(x) = 3(x+1)(x-2)^2$ ✓ ✓ ✓

1 S & A

1 correct factor

1 M – mark for long division or synthesis division

1 mark for factor $3x^2 - 12x + 12$

3 marks for the 3 factors $3(x+1)(x-2)$

Alternative solution 3

$R = h(2) = 3(2)^3 - 9(2)^2 + 12$

$= 24 - 36 + 12$

$= 0$ ✓

$\therefore x - 2$ is a factor of $h(x)$

$$x-2\overline{)3x^3-9x^2+0x+12}\quad 3x^2-3x-6 \quad ✓$$

$3x^3 - 6x^2$

$-3x^2 + 0x$ ✓

$-3x^2 + 6x$

$-6x + 12$

$-6x + 12$

0

$h(x) = (x-2)(3x^2 - 3x - 6)$

$\therefore h(x) = 3(x+1)(x-2)(x-2)$ ✓ ✓ ✓

$\therefore h(x) = 3(x+1)(x-2)^2$

Alternative solution 4

$$\begin{array}{c|cccc} 2 & 3 & -9 & 0 & 12 \\ & & 6 & -6 & -12 \\ \hline & 3 & -3 & -6 & 0 \end{array} \quad ✓ \; ✓ \; ✓$$

$\therefore f(x) = (x+1)(3x^2 - 3x - 6)$

$\therefore f(x) = 3(x+1)(x-2)^2$ ✓ ✓ ✓

Alternative solution 5

$$x-2\overline{)3x^3-9x^2+0x+12}\quad 3x^2-3x-6 \quad ✓$$

$3x^3 - 6x^2$

$-3x^2 + 0x$ ✓

$-3x^2 + 6x$

$-6x + 12$

$-6x + 12$

0 ✓

$\therefore f(x) = (x+1)(3x^2 - 3x - 6)$

$\therefore f(x) = 3(x+1)(x-2)^2$ ✓ ✓ ✓

(6)

1.4 $\dfrac{x+3}{x^2-7x+12} - \dfrac{x-3}{x^2-9}$

$= \dfrac{x+3}{(x-3)(x-4)} - \dfrac{x-3}{(x-3)(x+3)}$ √ √

$= \dfrac{(x+3)(x+3) - [(x-3)(x-4)]}{(x-3)(x+3)(x-4)}$ ✓ ✓

$= \dfrac{x^2 + 6x + 9 - (x^2 - 7x + 12)}{(x-3)(x+3)(x-4)}$

$= \dfrac{x^2 + 6x + 9 - x^2 + 7x - 12}{(x-3)(x+3)(x-4)}$ ✓

$= \dfrac{13x - 3}{(x-3)(x+3)(x-4)}$ ✓

½ factorising trinomial
½ factorising difference of two squares

2 marks for the equivalent fraction (fraction with the LCM)

1 mark for simplifying

1 mark for final answer

(5)

[22]

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QUESTION 2

SOLUTION | **EXPLANATION**

2.1

$$\frac{e^x}{e^{-x^2+6}} - e^{2x} = 0$$

$$\therefore e^{x+x^2-6} = e^{2x} \quad \surd$$

$$\therefore x + x^2 - 6 = 2x \quad \checkmark$$

$$\therefore x^2 - x - 6 = 0$$

$$\therefore (x-3)(x+2) = 0 \quad \checkmark$$

$$\therefore x - 3 = 0 \text{ or } x + 2 = 0$$

$$x = 3 \quad \checkmark \qquad x = -2 \quad \checkmark$$

½ mark for applying exponential rules and ½ mark for taking e^{2x} to the RHS

1 mark for equating
1 mark for factorisation
1 mark each for answers

Alternative solution 1

$$\frac{e^x}{e^{-x^2+6}} - e^{2x} = 0$$

$$\frac{e^x}{e^{-x^2+6}} = e^{2x}$$

$$\log \frac{e^x}{e^{-x^2+6}} = \log e^{2x} \quad \checkmark$$

$$\log e^x - \log e^{-x^2+6} = \log e^{2x}$$

$$x \log e - \left(-x^2+6\right)\log e = 2x \log e \quad \checkmark$$

$$\log e\left(x^2 + x - 6\right) = 2x \log e$$

$$x^2 - x - 6 = 0 \quad \checkmark$$

$$(x-3)(x+2) = 0$$

$$x = 3 \ \checkmark \ or \quad x = -2 \ \checkmark$$

Alternative solution 2

$$\frac{e^x}{e^{-x^2+6}} - e^{2x} = 0$$

$$\ln \frac{e^x}{e^{-x^2+6}} = \ln e^{2x} \quad \checkmark$$

$$\ln e^x - \ln e^{-x^2+6} = \ln e^{2x} \quad \checkmark$$

$$x - \left(-x^2+6\right) = 2x$$

$$x^2 - x - 6 = 0 \quad \checkmark$$

$$(x-3)(x+2) = 0$$

$$x = 3 \quad or \quad x = -2 \quad \checkmark$$

½ introduction of *log* on each side.

1 apply log laws

1 cancel *log e*

½ each factor

½ each CA

(5)

2.2 If $6^{2x} = 4^{x+2}$, prove that $x = \log_9 16$

$$6^{2x} = 4^{x+2}$$

$$\therefore 36^x = 4^x \times 16 \quad \checkmark$$

$$\therefore 4^x \times 9^x = 4^x \times 16 \quad \checkmark$$

$$\therefore \quad 9^x = 16 \quad \checkmark$$

$$\therefore \quad x = \log_9 16 \quad \checkmark$$

Alternative solution

$$6^{2x} = 4^{x+2}$$

$$\therefore 36^x = 4^x \times 16 \quad \checkmark$$

$$\therefore 4^x \times 9^x = 4^x \times 16 \quad \checkmark$$

$$\therefore \quad 9^x = 16 \quad \checkmark$$

$$\therefore \quad \log 9^x = \log 16$$

$$\therefore \quad x \log 9 = \log 16$$

$$\therefore x = \frac{\log 16}{\log 9}$$

$$\therefore x = \log_9 16 \quad \checkmark$$

1 mark applying exponential rules

1 M splitting the power

Equating to get $9^x = 16$ since $4^x \neq 0$

Applying the definition of logs (revert from exponential form to log form)

(4)

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2.3 $$\frac{\sqrt{(a-b)^3}\times\sqrt{a-b}\times\sqrt{a+b}}{\sqrt[3]{(a-b)^5}\times(a^2-b^2)^{\frac{1}{3}}}$$

$$=\frac{(a-b)^{\frac{3}{2}}\times(a-b)^{\frac{1}{2}}\times(a+b)^{\frac{1}{2}}}{(a-b)^{\frac{5}{3}}\times(a-b)^{\frac{1}{3}}(a+b)^{\frac{1}{3}}}$$

$$=(a-b)^{\frac{3}{2}+\frac{1}{2}-\frac{5}{3}-\frac{1}{3}}\times(a+b)^{\frac{1}{2}-\frac{1}{3}}$$

$$=(a+b)^{\frac{1}{6}}(a-b)^{0}$$

$$=(a+b)^{\frac{1}{6}}$$

½ M each for applying surd laws by writing surds in exponential form

½ each for correct sum of powers

1 mark for final answer (remember $(a-b)^0=1$ and 1 is the identity element for multiplication)

(5)
[14]

QUESTION 3

	SOLUTION	EXPLANATION	
3.1.	Let x be the amount Luvuyo win		
	: $Luvuyo = x$ $Sbusiso = x + 80$ ✓	1 S (indicating what x represents)	
	$Vusi = \frac{1}{2}(x + 80)$		
	$x + x + 80 + \frac{1}{2}(x + 80) = 710$ ✓		
	$2x + 80 + \frac{x}{2} + 40 = 710$	1 SF	
	$2x + \frac{x}{2} = 710 - 120$		
	$\frac{4x + x}{2} = 590$		
	$5x = 1180$		
	$x = 236$		
	$Luvuyo\ won\ R236$ ✓		
	$Sbusiso = x + 80 = 236 + 80 = R316$ ✓		
	$Vusi = \frac{1}{2}(x + 80) = \frac{1}{2}(236 + 80) = R158$ ✓	1 mark each for final answer	(5)
3.2.	$P = \frac{A}{\left(1 + \frac{i}{m}\right)^{n \times m}}$	1 cross multiplication	
	$\left(1 + \frac{i}{m}\right)^{n \times m} = \frac{A}{P}$ ✓	1 taking the $n \times m$ root	
	$1 + \frac{i}{m} = \sqrt[n \times m]{\frac{A}{P}}$ ✓		
	$i = m\left(\sqrt[n \times m]{\frac{A}{P}} - 1\right)$ √ √	½ transposing 1 ½ multiplying by m	(3)

3.3. $x+y=3$ .... (1)

$2x^2+4xy-y=15$ ..(2)

Make y the subject of the formula in (1

$\therefore y=-x+3$ √

Replace y by $-x+3$ in (2):

$\therefore 2x^2+4x(-x+3)-(-x+3)=15$ √

$\therefore 2x^2-4x^2+12x+x-3-15=0$

$\therefore -2x^2+13x-18=0$ ✓

$\therefore (2x-9)(x-2)=0$ ✓

$\therefore x=\frac{9}{2}$ or $x=2$

If $x=\frac{9}{2}$ then $y=-\left(\frac{9}{2}\right)+3$

$y=-\frac{3}{2}$ ✓

If $x=2$ then $y=-(2)+3$

$=1$ ✓

Alternative solution

$x+y=3$ ...(1)

$2x^2+4xy-y=15$ ..(2)

$x=-y+3$ √

$\therefore 2(-y+3)^2+4y(-y+3)-y-15=0$ √

$\therefore 2y^2-12y+18-4y^2+12y-y-15=0$

$\therefore 2y^2+y-3=0$

$\therefore (2y+3)(y-1)=0$

$\therefore y=-\frac{3}{2}$ ✓ or $y=1$ ✓

If $y=-\frac{3}{2}$ then $x=-\left(-\frac{3}{2}\right)+3$

$=\frac{9}{2}$ ✓

or if $y=2$ then $x=-(1)+3$

$=2$ ✓

½ mark for making y/x the subject of the formula

½ SF

1 mark for each correct x value

1 mark for each correct y value

(5)
[13]

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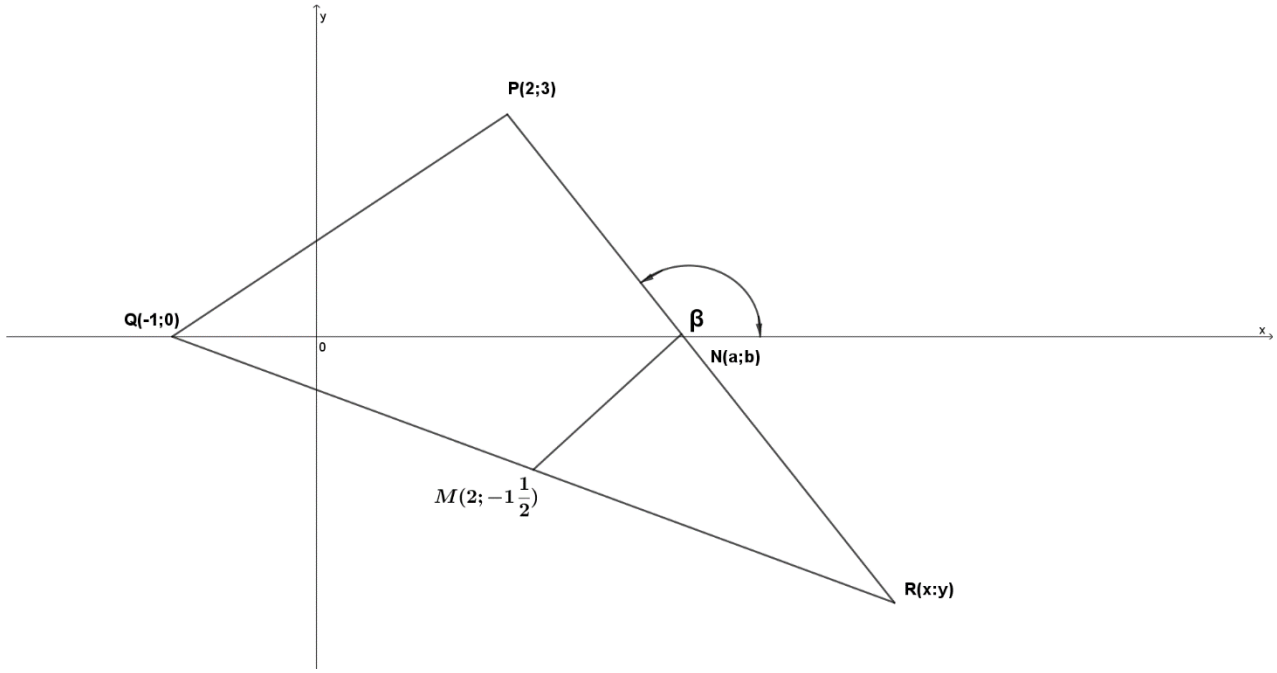
QUESTION 4



SOLUTION	EXPLANATION
4.1	
$x=\frac{x_1+x_2}{2}$ $\quad y=\frac{y_1+y_2}{2}$	
$2=\frac{-1+x_2}{2}$ √ $\quad -\frac{3}{2}=\frac{0+y_2}{2}$ √	½ for each correct substitution (SF)
$4=-1+x_2$ $\quad -\frac{6}{2}=0+y_2$	1 mark each for the co-ordinates of $R(5;-3)$
$x_2=5$ ✓ $\quad y_2=-3$ ✓	
$\therefore \text{R}(5:-3)$	(3)
4.2	
Midpoint N : $=\left(\frac{x_1+x_2}{2};\frac{y_1+y_2}{2}\right)$ √ √	½ mark for each correct substitution
$=\left(\frac{2+5}{2};\frac{3-3}{2}\right)$	
$=\left(\frac{7}{2};0\right)$ ✓	1 mark for correct answer
CA | Follow up from 4.1	(2)

4.3

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(-1-2)^2 + (0-3)^2} \checkmark$$
$$= \sqrt{9+9}$$
$$= \sqrt{18}$$
$$= 3\sqrt{2} \checkmark$$

1 mark for correct substitution

1 mark for correct answer (in simplest surd form)

(2)

4.4

$$M_{MN} = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{0 - \left(-\frac{3}{2}\right)}{\frac{7}{2} - 2} \checkmark$$
$$= \frac{3}{2} \times \frac{2}{3}$$
$$= 1 \checkmark$$

$$y - y_1 = m(x - x_1)$$
$$y + \frac{3}{2} = 1(x - 2) \checkmark$$
$$y = x - 2 - \frac{3}{2}$$
$$y = x - \frac{7}{2} \checkmark$$

CA	Follow up from 4.2

1 mark each for correct substitutions on both

1 mark for correct gradient of 1

1 SF

1 mark for writing the straight line in the form $y = mx + c$.

(4)

4.5

$$M_{PR} = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{-3-3}{5-2} \checkmark$$
$$= \frac{-6}{3}$$
$$= -2 \checkmark$$

$$\beta = \tan^{-1} m$$
$$= \tan^{-1}(-2) \checkmark$$
$$\therefore \beta = 180° - 63,435°$$
$$= 116,565° \checkmark$$

CA	Follow up from 4.1

2 marks for determining the gradient

1 SF

1 mark for the correct obtuse angle

(4)

4.6

$$M_{PR} = -2$$
$$\therefore -2 \times m = -1$$
$$m = \frac{1}{2} \checkmark$$

$$y - y_1 = m(x - x_1)$$
$$y - 3 = \frac{1}{2}(x - 2) \checkmark$$
$$y = \frac{1}{2}x - 1 + 3$$
$$y = \frac{1}{2}x + 2 \checkmark$$

CA	Follow up from 4.5 for $\perp$ line

1 mark for determining the gradient of a perpendicular line

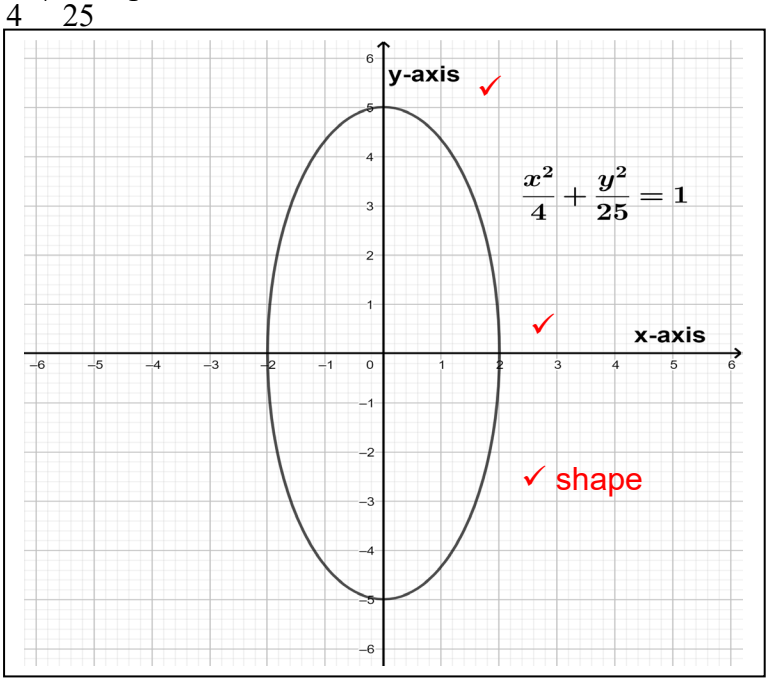
1 SF

1 mark for writing the straight line in the form $y = mx + c$.

(3)

[18]

QUESTION 5

	SOLUTION	EXPLANATION
5.1	$100x^2+16y^2=400$ $\frac{100x^2}{400}+\frac{16y^2}{400}=\frac{400}{400}$ $\frac{x^2}{4}+\frac{y^2}{25}=1$	1 for x-intercept 1 for y-intercept 1 for shape
		(3)
5.2	$x=-\frac{b}{2a}=-\frac{(-3)}{2(1)}=\frac{3}{2}$ $y=\left(\frac{3}{2}\right)^2-3\left(\frac{3}{2}\right)+2$ $=\frac{9}{4}-\frac{9}{2}+2$ $=-\frac{1}{4}$ $TP=\left(\frac{3}{2};-\frac{1}{4}\right)$	1 for x value of axis of symmetry ($x=\frac{3}{2}$) 1 for y value of axis of symmetry ($y=-\frac{1}{4}$) Award full marks if answer not in a coordinate form
	Alternative solution 1 $y=x^2-3x+2$ $y=\left(x-\frac{3}{2}\right)^2-\left(-\frac{3}{2}\right)^2+2$ $y=\left(x-\frac{3}{2}\right)^2-\frac{9}{4}+\frac{8}{4}$ $y=\left(x-\frac{3}{2}\right)^2-\frac{1}{4}$ $\therefore TP=\left(\frac{3}{2};-\frac{1}{4}\right)$	

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Alternative solution 2

$f(x) = x^2 - 3x + 2$

$\therefore f'(x) = 2x - 3$

For TP, let $f'(x) = 0$

$\therefore 2x - 3 = 0$

$\therefore \quad x = \frac{3}{2}$

If $x = \frac{3}{2}$, then $f(x) = \left(\frac{3}{2}\right)^2 - 3\left(\frac{3}{2}\right) + 2$

✓ ✓ $= -\frac{1}{4}$

$TP = \left(\frac{3}{2}; -\frac{1}{4}\right)$

(2)

5.3 $y = \frac{\sqrt{x}}{2} + \frac{1}{3x^3} - \pi x$

$= \frac{1}{2}x^{\frac{1}{2}} + \frac{1}{3}x^{-3} - \pi x$ √ √

$\frac{dy}{dx} = \frac{1}{2} \times \frac{1}{2}x^{\frac{1}{2}-1} + \frac{1}{3} \times -3x^{-3-1} - \pi x^{1-1}$

$= \frac{x^{-\frac{1}{2}}}{4} - x^{-4} - \pi(1)$ ✓ ✓ ✓

$= \frac{1}{4\sqrt{x}} - \frac{1}{x^4} - \pi$ √ √

½ mark for each term in the preparation step

1 mark for each derivative

½ mark for writing each derivative into positive and in surd form

(5)

5.4 $f(x) = -x^3 - 2x^2$

$f'(x) = -3x^2 - 4x$ ✓

For TP's, let $f'(x) = 0$

$\therefore -3x^2 - 4x = 0$ ✓

$\therefore x(-3x - 4) = 0$

$\therefore x = 0$ or $-3x - 4 = 0$

$\therefore x = 0$ *or* $-3x = 4$

$x = -\frac{4}{3}$ ✓

If $x = -\frac{4}{3}$, then $f(x) = -\left(-\frac{4}{3}\right)^3 - 2\left(-\frac{4}{3}\right)^2$

$= -\frac{32}{27}$ *or* $-1{,}185$ ✓

$\therefore TP = \left(-\frac{4}{3}; -\frac{32}{27}\right)$

1 mark for derivative

1 mark for equating to zero (gradient of tangent at max point = 0)

1 mark for the correct $x-coordinate$

1 mark for the correct $y-coordinate$

Award full marks if answer not in a coordinate form (4)

[14]

QUESTION 6

	SOLUTION	EXPLANATION
6.1	$\text{LHS} = \text{cosec}\,B \times \sec B \times -\cot B$	
	$= \frac{1}{\sin B} \times \frac{1}{\cos B} - \frac{\cos B}{\sin B}$ ✓	1 mark for applying trigonometrical identities
	$= \frac{1 - \cos^2 B}{\sin B \cos B}$ ✓	
	$= \frac{\sin^2 B}{\sin B \cos B}$ ✓	1 mark for simplifying two steps
	$= \frac{\sin B}{\cos B}$	
	$= \tan B$ ✓	1 mark for applying trigonometrical identities
	$= RHS$	1 mark for correct answer (4)

6.2 $4\cos^2 x + 4\sin^2 x = 5 - \tan x$

$4\left(\cos^2 x + \sin^2 x\right) + \tan x = 5$ ✓

$4(1) + \tan x = 5$

$\tan x = 1$

$x = \tan^{-1}(1)$ ✓

$x = 45°$ ✓

$\therefore x = 180° + 45°$

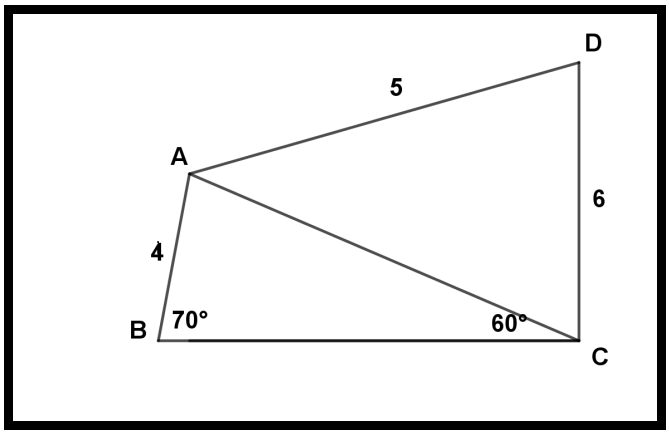
$= 225°$ ✓

1 mark for factorising

1 mark for $x = \tan^{-1}(1)$

1 mark each for $x = 45°$ and $x = 225°$ (4)

6.3



6.3.1 In ΔABC:

$\frac{AC}{\sin 70°} = \frac{4}{\sin 60°}$ ✓

$\therefore AC = 4,34cm$ ✓

1 F

1 SF (2)

6.3.2 $4,34^2 = 5^2 + 6^2 - 2(5)(6)\cos D$ ✓

$18,836 = 25 + 36 - 60\cos D$

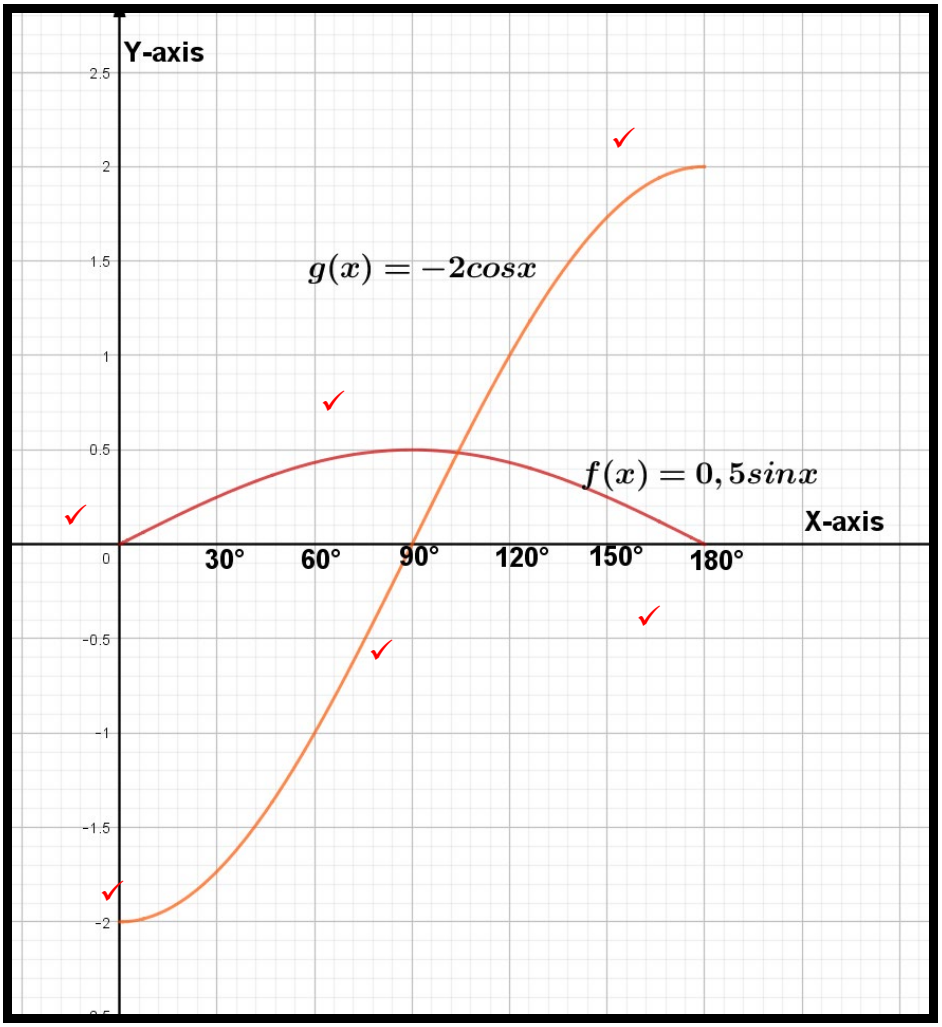
$0,703 = \cos D$ ✓

$45,35° = \hat{D}$ ✓

1 F
1 SF

1 mark for answer (3)

6.4



2 marks for the y-intercepts (starting points)

2 marks for the turning points of both graphs $(90^{\circ};0,5)$ and $(180^{\circ};2)$

2 marks for the $x-$intercepts

(6)
[19]

TOTAL: 100

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