Exemplar examination paper

Calculation questions

1. An aircraft takes off from Grand Central Airport in a direction 30° East of North. The airspeed of the plane is 250 km/h and it has a strong tail wind of 85 km/h 10° West of North.

   Calculate:

   1.1 the resultant velocity of the aircraft and the direction of the flight.

   1.2 the distance covered by the plane after 45 minutes in the air. (5)

2. A tennis ball canon shoots a ball from ground level over the net at 1,1 m and it lands back in the court 24 m from the cannon.

   Calculate:

   1.1 The vertical velocity of the ball when it leaves the cannon. (2)

   1.2 The total flight time. (1)

   1.3 The horizontal velocity of the ball. (1)

   1.4 The angle of the cannon. (1)

   1.5 The initial speed of the ball when it leaves the cannon. (1)

3. A man cycling on a bicycle accelerates from standstill to 43,2 km/h in 10 seconds. His wheel size is 70 cm.

   Calculate:

   2.1 His angular velocity after 10 seconds. (2)

   2.2 His angular acceleration in this period. (1)

   2.3 The angular displacement in this period. (1)

   3.4 His linear displacement in this time. (1)

4. An industrial grinding wheel weighs 10 kg and has a 45-cm diameter.

   Calculate:

   4.1 The torque required to accelerate it from standstill to 2 500 rpm in 20 seconds. (3)

   4.2 The power required to accelerate the wheel. (1)

5. The brakes of a truck travelling at a speed of 66 km/h fail and it starts running down a hill that is 2-km long with a 1:15 gradient. A steep hill thereafter would bring it to a stop. The truck weighs 4,9 tonnes and has internal friction of 1 600 N.

   Calculate the following:

   5.1 The acceleration of the truck (4)

   5.2 The top speed of the truck (2)

   5.3 The time it takes to travel this distance (1)

   5.4 The friction loss in the truck (2)

   5.5 The efficiency of the truck's running gear (2)

6. A 4-m long beam, ABCDE, is divided into 4 sections of 1 meter each. The beam is simply supported at A and C with a 40-kN point load at B and a 20-kN point load at E.
   - 6.1 Draw the beam and calculate the reaction forces at A and D. (4)
   - 6.2 Calculate the bending moments at B, C and D. (3)
   - 6.3 Draw the bending moment and shear force diagrams and indicate the main values on the diagrams. (4)
   - 6.4 Determine the maximum bending moment and its position in the beam. (2)
   - 6.5 Draw an exaggerated profile of the beam and determine its point of inflection. (2)
7. A foot-operated Arbor press has a 3-cm diameter master cylinder with a stroke length of 10 cm. It has 7% slip and drives a 6-cm diameter ram to apply its pressure. The foot pedal provides 4 times a person's body weight in mechanical advantage onto the plunger of the master cylinder.
   - 7.1 What is the stroke length of the ram for a single stroke of the master cylinder? (4)
   - 7.2 What mass would the system by able to lift if the operator weighs 60 kg? (4)
8. A 1,2-tonne mass hangs on a 1,5-m long, 12-mm diameter steel rod. The rod stretches 0,8 mm under the weight.

   Calculate:
   - 8.1 The tensile stress (2)
   - 8.2 The strain (2)
   - 8.3 Young's modulus (2)
9. Two cubic meters (2 m^3) of nitrogen gas at 220°C and 500 kPa is cooled down to 35 °C. What is the volume of the cold gas at atmospheric pressure? (3)
10. In an isochoric process, 100 moles of ammonia at 300 kPa and 22°C flashes into a drum at a pressure of 120 kPa.

    Calculate:
    - 10.1 The final temperature (2)
    - 10.2 The work done (1)
    - 10.3 The change in internal energy (3)
    - 10.4 The energy flow (1)
    - 10.5 The volume (2)
    - 10.6 Draw the PV diagram (3)

Total: 75 marks

Theory questions

1. What are the units of measurement for:
   a. Linear displacement
   b. Angular displacement
   c. Linear velocity
   d. Angular velocity
   e. Linear acceleration
   f. Angular acceleration (6)

2. Give the units of measurement and explain the concept of:
   a. Work done
   b. Power
   c. Pressure
   d. Torque
   e. Kinetic energy
   f. Potential energy
   g. Stress
   h. Strain
   i. Heat (10 × 2)

3. Explain what is meant by the following terms:
   a. Trajectory
   b. Negative acceleration
   c. Friction coefficient
   d. Shearing force
   e. Bending moment
   f. Simply supported beam
   g. Hydraulics
   h. Coefficient of linear expansion
   i. Specific heat capacity (10)

4. State:
   a. Newton's First law
   b. Newton's Second law
   c. Newton's Third law
   d. Law of the conservation of energy (4)

5. Define:
   a. Pascal's law
   b. Boyle's law
   c. Hooke's Law
   d. Charles' law
   e. Young's modulus (5)

6. Describe and explain the function of a:
   a. Hydraulic accumulator
   b. Hydraulic press
   c. Hydraulic pump
   d. Ram in hydraulics

7. Name TWO differences between *speed* and *velocity*. (2)

8. Name THREE characteristics of a liquid or fluid. (3)

9. Name THREE types of hydraulic accumulators. (3)

10. Name FOUR functions of hydraulic accumulators. (4)

11. Name THREE types of hydraulic pumps and explain how each is used in industry. (3 × 2)

12. Name THREE types of stresses that can be found in materials. (3)
13. Explain the difference between the Kelvin scale and the Celsius scale. (2)
14. What is the numeric ratio between the coefficients of area expansion and volume expansion? (2)
15. What is the difference between absolute zero and absolute temperature? (2)

Total: 62 marks

Exemplar examination paper memorandum

Please note that the answers to theory questions have not been provided but can be accessed by students and lecturers alike in the Student Book.

Calculation questions

1. **Direction east:**

$$v_{RE} = v_{PE} + v_{WE}$$
$$= 250\sin 30° - 85\sin 30°$$
$$= 82{,}5 \text{ km/h East}$$

Direction north:

$$v_{RN} = v_{PN} + v_{WN}$$
$$= 250\cos 30° + 85\cos 30°$$
$$= 290{,}1 \text{ km/h North}$$

Resultant velocity:

$$v_R = \sqrt{82{,}5^2 + 290{,}1^2}$$
$$= 301 \text{ km/h}$$

Resultant direction:

$$\theta = \tan^{-1}\frac{82{,}5}{290{,}1}$$
$$= 15{,}9° \text{ East of North}$$

Distance after 45 minutes:

$$d_R = \left(\frac{45}{60}\right)301 = 225{,}8 \text{ km}$$

2. **Initial vertical velocity:**

$$v_y^2 = v_{yi}^2 + 2a\,d_y$$
$$0^2 = v_{yi}^2 - 2(9{,}8)1{,}1$$
$$v_{yi} = \sqrt{21{,}56} = 4{,}643 \text{ m/s}$$

Total flight time:

$$v_{yf} = v_{yi} + at$$
$$-4{,}643 = 4{,}643 - 9{,}8t$$
$$t = 0{,}948 \text{ s}$$

Horizontal velocity:

$$d_x = v_x t$$

$$\therefore v_x = \frac{d}{t}$$

$$= \frac{24}{0{,}948} = 25{,}33 \text{ m/s}$$

The angle is:

$$\tan\theta = \frac{v_{yi}}{v_{xi}} = \frac{4{,}643}{25{,}33} = 0{,}183$$

$$\theta = \sin^{-1}0{,}183 = 10{,}4°$$

Initial speed:

$$v_R^2 = v_{xi}^2 + v_{yi}^2$$

$$v_R = \sqrt{25{,}33^2 + 4{,}643^2} = 25{,}75 \text{ m/s}$$

3. $v_f = 43{,}2 \times \frac{1\,000}{3\,600} = 12 \text{ m/s}$

$$\omega_f = \frac{v_f}{r} = \frac{12}{0{,}7} = 17{,}14 \text{ rad/s}$$

$$\alpha = \frac{\omega_f - \omega_i}{t} = \frac{17{,}14 - 0}{10} = 1{,}71 \text{ rad/s}^2$$

$$\theta = \frac{\omega_f + \omega_i}{2} \cdot t$$

$$= \frac{17{,}14}{2} \cdot 10 = 85{,}7 \text{ rad}$$

$$d = \theta r = 85{,}7 \times 0{,}7 = 60 \text{ m}$$

4. $\omega_f = \frac{2\pi n_f}{60}$

$$= \frac{2\pi \times 2\,500}{60} = 261{,}8 \text{ rad/s}$$

$$\alpha = \frac{\omega_f - \omega_i}{\Delta t}$$

$$= \frac{261{,}8 - 0}{20} = 13{,}09 \text{ rad/s}^2$$

$$\tau = mr^2\alpha$$

$$= 10 \times 0{,}45^2 \times 13{,}09 = 26{,}51 \text{ Nm}$$

$$P = \tau\omega$$

$$= 26{,}51 \times 261{,}8 = 6\,940 \text{ W}$$

5. a. **Acceleration of the truck:**

$$\theta = \tan^{-1}\left(\frac{1}{15}\right) = 3{,}81°$$

$$v_i = 66 \times \frac{1\,000}{3\,600} = 18{,}33 \text{ m/s}$$

$$\Sigma F = F_x - f$$

$$= mg\sin\theta - f$$

$$= 4\,900 \times 9{,}8 \times \sin 3{,}81° - 1\,600$$

$$= 1\,594 \text{ N}$$

$$a = \frac{F}{m}$$

$$= \frac{1\,594}{4\,900} = 0{,}325 \text{ m/s}^2$$

b. **Top speed:**

$$v_f^2 = v_i^2 + 2ad$$

$$v_f^2 = 18{,}33^2 + 2 \times 0{,}325 \times 2\,000 = 1\,638$$

$$\therefore\ v_f = 40{,}47 \text{ m/s} \left(= 146 \text{ km/h} \right)$$

c. **The time taken:**

$$v_f = v_i + at$$

$$t = \frac{v_f - v_i}{a}$$

$$= \frac{40{,}47 - 18{,}33}{0{,}325} = 68{,}0 \text{ s}$$

d. **Friction losses:**

$$h = 2\,000 \times \sin 3{,}81° = 133 \text{ m}$$

$$\% \text{ losses} = \frac{\Sigma E_{\text{losses}}}{\Sigma E_{\text{in}}} \times 100\,\%$$

$$= \frac{fd}{mgh + \frac{1}{2}mv^2} \times 100\,\%$$

$$= \frac{1\,600 \times 2\,000 \times 100}{4\,900 \times 9{,}8 \times 133 + \frac{1}{2} \times 4\,900 \times 18{,}33^2} = 44{,}4\%$$

e. **Efficiency of truck:**

$$\eta = \frac{E_{k.\text{bot}}}{E_p + E_{k.\text{top}}} \times 100\,\%$$

$$= \frac{\frac{1}{2} \times 4\,900 \times 40{,}47^2 \times 100}{4\,900 \times 9{,}8 \times 133 + \frac{1}{2} \times 4\,900 \times 18{,}33^2}$$

$$= 55{,}6\%$$

6. $\circlearrowleft \Sigma M = \circlearrowright \Sigma M$

$$F_C d_C = F_B d_B + F_E d_E$$

$$F_C \times 2 = 40 \times 1 + 15 \times 4$$

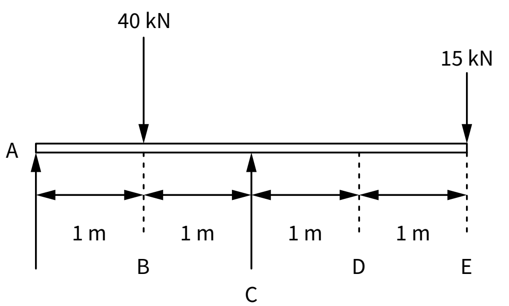
$$\therefore F_C = 50 \text{ kN}$$

$$\uparrow \Sigma F = \downarrow \Sigma F$$

$$F_A + F_C = F_B + F_E$$

$$\therefore F_A = 40 + 15 - 50$$

$$= 5 \text{ kN}$$



$$M_A = M_E = 0$$

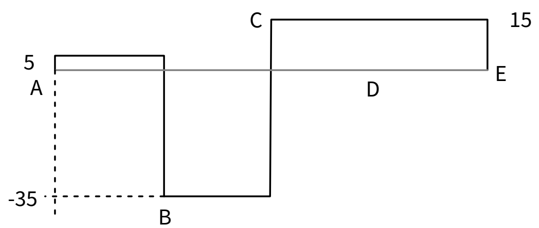
$$M_B = 5 \times 1 = 5 \text{ kNm}$$

$$M_C = 5 \times 2 - 40 \times 1 = -30 \text{ kNm}$$

$$M_D = 5 \times 3 - 40 \times 2 + 50 \times 1$$

$$= -15 \text{ kNm}$$

The maximum bending moment is at point C and M_C = 30 kNm (CCW).

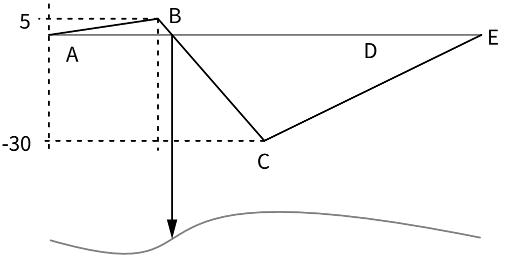


The point of inflection is between B and C.

$$\frac{IB}{5} = \frac{1}{35} \therefore$$

$$IB = 0{,}143 \text{ m}$$

$$\Rightarrow AI = 1{,}143 \text{ m}$$



7. **Master cylinder volume:**

$$V_m = \frac{\pi d^2}{4} l$$

$$= \frac{\pi \times 0{,}03^2}{4} \times 0{,}10$$

$$= 7{,}07 \times 10^{-5} \text{ m}^3$$

Volume transferred to the ram:

$$V_s = \eta_V V_m$$

$$= 93\,\% \times 7{,}07 \times 10^{-5}$$

$$= 6{,}57 \times 10^{-5} \text{ m}^3$$

Ram lift:

$$V_s = A_s l_s$$

$$\therefore l_s = \frac{4}{\pi d^2} \times V_s$$

$$= \frac{4}{\pi \times 0{,}06^2} \times 6{,}57 \times 10^{-5}$$

$$= 23{,}25 \times 10^{-3} \text{ m}$$

Force on the plunger:

$$F_m = 4 \times mg$$

$$= 4 \times 60 \times 9{,}8 = 2\ 352 \text{ N}$$

System pressure:

$$P = \frac{F_m}{A_m} = \frac{4}{\pi \cdot d^2} \times F_m$$

$$= \frac{4}{\pi \cdot 0{,}03^2} \times 2\ 744 = 3{,}33 \times 10^6 \text{ Pa}$$

Force in the ram:

$$F_s = P \cdot A_s$$

$$= 3.33 \times 10^6 \times \frac{\pi \cdot 0{,}06^2}{4} = 9\ 408 \text{ N}$$

In terms of mass:

$$m = \frac{F}{g} = \frac{9\ 408}{9{,}8} = 960 \text{ kg}$$

8. a. **Tensile stress:**

$$\sigma_T = \frac{F}{A} = mg \times \frac{4}{\pi d^2}$$

$$= \frac{4 \times 1\ 200 \times 9{,}8}{\pi \cdot 0{,}012^2} = 1{,}04 \times 10^8 \text{ Pa}$$

b. **Strain:**

$$\varepsilon = \frac{\Delta l}{l}$$
$$= \frac{0,8 \times 10^{-3}}{1,5} = 5,33 \times 10^{-4}$$

c. **Young's modulus:**

$$E = \frac{\sigma}{\varepsilon}$$
$$= \frac{1,04 \times 10^{8}}{5,33 \times 10^{-4}} = 195 \times 10^{9} \text{ Pa}$$

9. **Moles of nitrogen:**

$$n = \frac{PV}{RT}$$
$$= \frac{500\,000 \times 2}{8,314 \times (220 + 273,1)} = 243,9 \text{ mol}$$

Calculate the new volume:

$$V = \frac{nRT}{P}$$
$$= \frac{243,9 \times 8,314 \times (35 + 273,1)}{101\,325} = 6,17 \text{ m}^3$$

10. **Final temperature:**

$$\frac{T_1}{P_1} = \frac{T_2}{P_2}$$
$$\therefore T_2 = T_1 \frac{P_2}{P_1}$$
$$= (22 + 273,1)\frac{120\,000}{300\,000} = 118 \text{ K}$$

Work done:

$$W = P\Delta V = 0$$

Change in internal energy:

$$\Delta U = nC_v\Delta T$$
$$C_v = \frac{5}{2}R$$
$$\therefore \Delta U = \frac{5}{2}nR\Delta T$$
$$= \frac{5}{2} \cdot 100 \cdot 8,314 \cdot (118 - 295,1)$$
$$= -368 \text{ kJ}$$

Energy flow:

$$Q = W + \Delta U$$
$$= 0 - 368 = -368 \text{ kJ}$$

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The volume:

$$V = \frac{nRT_i}{P_i}$$

$$= \frac{100 \times 8{,}314 \times 295{,}1}{300\ 000}$$

$$= 0{,}818\ m^3$$