Exemplar examination paper

Question 1: Thick cylinders

Two hollow cylinders were shrunk together to form a compound cylinder with an inner diameter of 100 mm and an outer diameter of 400 mm. This caused an intermediate pressure at the common diameter of 200 mm. After the cylinders were shrunk together, an internal pressure of 30 MPa was applied to the compound cylinder, causing the resultant hoop stress at the inner diameter to reach 46 MPa (compressive).

Calculate the following:

1.1 The resultant stresses in the inner cylinder at 200 mm
1.2 The resultant stresses in the outer cylinder at 200 mm and 400 mm respectively
1.3 Sketch a stress distribution diagram to indicate the magnitude and nature of the resultant stresses through the compound cylinder walls.

Final answers for solutions:
M = 10^6 and $k = 10^3$ and G = 10^9 , where M, k and G are not part of an equation.

Question 2: Tension in cables

The supports of a suspension bridge are 36 m apart and the on the same level. The sag of the cables is 3 m. The roadway has a total weight of 3 024 kN.

Calculate the following:

2.1 The weight per metre carried by each of the two cables
2.2 The minimum and maximum tensions in each cable
2.3 The diameter required for the cable if the ultimate tensile stress for the cable material is limited to 320 MPa (use a safety factor of 8)
2.4 The tension in the cable 10 m from the support, measured horizontally.

Question 3: Combined bending and twisting of shafts

A solid shaft with a diameter of 80 mm is subjected to a maximum torque of 4 kNm as well as a bending moment. The shear stress in the shaft is limited to 50 MPa and the principal stress is limited to 75 MPa.

Calculate the following:

3.1 The maximum bending moment by considering the shear stress
3.2 The maximum bending moment by considering the principal stress
3.3 The maximum bending moment allowed, and provide a reason
3.4 The actual shear stress in the shaft.

Question 4: Bending and deflection of beams

A steel pipe, having an inside diameter equal to half of the outside diameter, is used as a cantilever with a length of 2,5 m. It carries a uniformly distributed load of 20 kNm over the first 1,25 m from the fixed end, as well as a concentrated load at the free end. The deflection at the free end is limited to 7 mm. The modulus of elasticity for the material is 200 GPa.

Calculate the following:

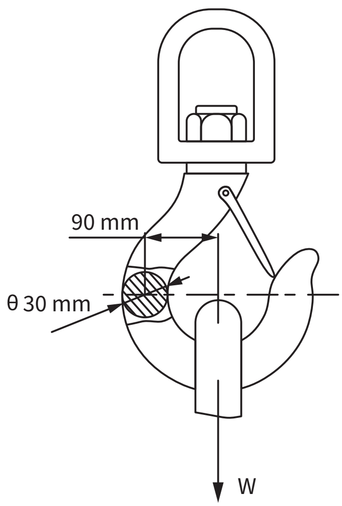
4.1 The required dimensions for the pipe

4.2 Choose the lightest taper flange I-profile that can replace the pipe for the same deflection limit.

4.3 The maximum bending stress if the selected I-profile is used.

Question 5: Combined bending and direct stresses

A crane hook has a circular cross section with a diameter of 30 mm as shown below. The distance that the load is applied from the centroid is 90 mm. The tensile stress in the material of the crane hook may not exceed 90 MPa.



Crane hook

Calculate the following:

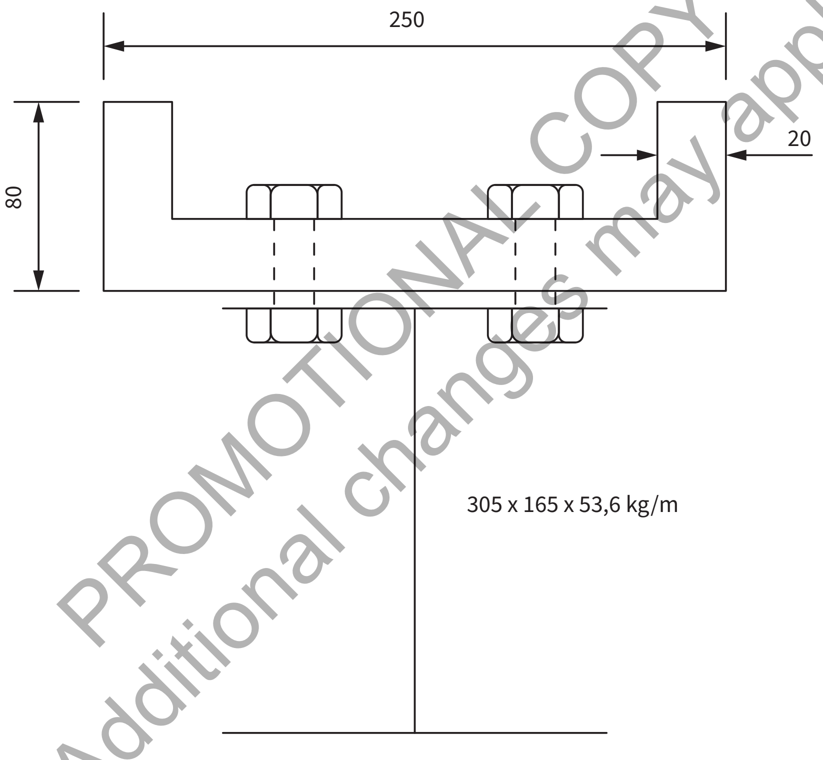
5.1 The maximum mass that may be lifted

5.2 The minimum stress in the hook in magnitude and nature.

Question 6: Shear stress in beams

A built-up beam consists of a hot-rolled parallel flange I-beam with dimensions of 305 × 165 × 53,6 kg/m, and a channel bolted to the I-beam by two rows of bolts. The height of the channel is 250 mm and the breadth is 80 mm, with a thickness of 20 mm right through. The build-up beam is subjected to a shear force of 100 kN. The web is connected to the flange by means of two 17-mm diameter bolts and the shear stress in the bolts is 80 MPa.

Calculate the spacing of the bolts.



Built-up beam

Question 7: Close-coiled helical springs

A maximum load of 45 N compresses a coil spring to a solid length of 45 mm. The stiffness of the spring is 900 Nm and the maximum shear stress in the wire is 120 MPa. G = 40 GPa.

Calculate:

7.1 The wire diameter of the spring

7.2 The diameter of the spring

7.3 The number of coils.

Question 8: Transformation of stress

At a point in a material, the principal stresses are 40 MPa and 100 MPa respectively, and both stresses are tensile. Turn the element 15° anti-clockwise to the plane on which the principal stresses act.

Calculate:

8.1 The normal stresses on this plane for the *x*-face

8.2 The shear stress on this plane

8.3 Use Mohr's circle to verify the answers on the stress circle.

8.4 Use the circle and determine the maximum shear stress as well as the normal stress on the X-face if the element is turned 90° in the same direction to the plane above.

Question 9: Forces in structural frameworks

The legs of a tripod are each 6 m long and are placed in such a way as to form an equilateral triangle ABC with sides 5 m on the ground. The tripod supports a load of 6 kN from the apex.

Use a scale of 1 cm = 1 m for the space diagram.
Use a scale of 1 cm = 1 kN for the vector diagram.

9.1 Draw the side- and top views of the tripod to the given scale to determine the apex.

9.2 Draw the vector diagrams to the given scale and determine the force in each leg. Redraw the following table in your answer book and tabulate the answers.

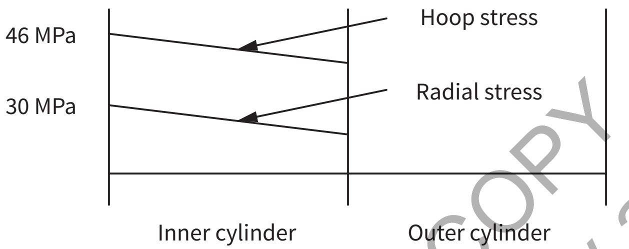
Member	Magnitude

9.3 Calculate the minimum coefficient of friction required between the legs and the ground to prevent slipping.

Exemplar examination paper memorandum

Question 1: Thick cylinders

1. 1.1 Resultant stresses in the inner cylinder at 200 mm



At $d = 100$; $\sigma_R = 30\text{M} = a + \frac{b}{0{,}1^2}$

$\therefore 30\text{M} = a + 100b \ldots$ ①

and at $d = 1i = a - \frac{b}{0{,}1^2}$

$\therefore 46\text{M} = a - 100b \ldots$ ②

① + ②: $\therefore 76\text{M} = 2a$

$\therefore a = 38\text{M} \ldots$ ③

Substitute ③ into ①: $\therefore 30\text{M} = 38\text{M} + 100b$

$\therefore b = -0{,}08\text{M}$

Stresses at D = 200 mm for inner cylinder

Hoop stress: Dc = 200 $\sigma_H = 38\text{M} - \frac{-0{,}08\ \text{M}}{0{,}2^2} = 40$ MPa

and Radial stress: Dc = 200 $\sigma_R = 38\text{M} + \frac{-0{,}08\ \text{M}}{0{,}2^2} = 36$ MPa

1.2 The resultant stresses in the outer cylinder at 200 mm and 400 mm

At D = 400; $\sigma_R = 0 = a + \frac{b}{0{,}4^2}$

$\therefore a = -6{,}25b \ldots$ ④

At Dc = 200: $\sigma_R = 36\text{M} = a + \frac{b}{0{,}2^2} \ldots$ ⑤

$\therefore$ Substitute (4) into (5) $\therefore 36\text{M} = -6{,}25b + 25b$

$\therefore b = 1{,}92\text{M}$

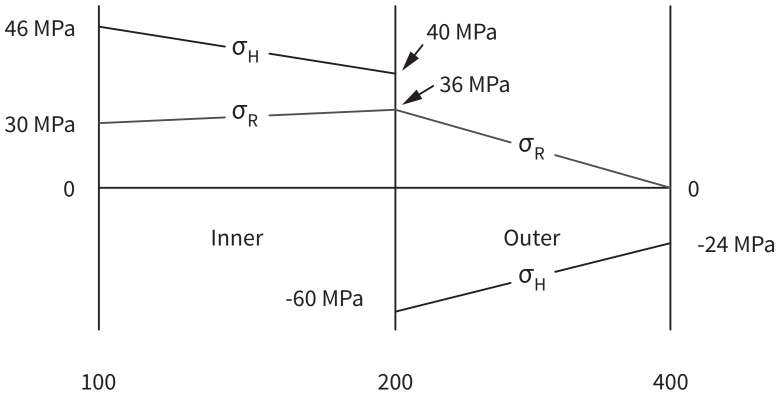
$\therefore a = -12\text{M}$

at $d = 200$; $\sigma_H = -12\text{M} - \frac{1{,}92\ \text{M}}{0{,}2^2} = -60$ MPa (T) at $d = 400$; $\sigma_H = -12M - \frac{1{,}92\ M}{0{,}4^2} = -24$ MPa (T)

Radial stress at 200 = 36 MPa

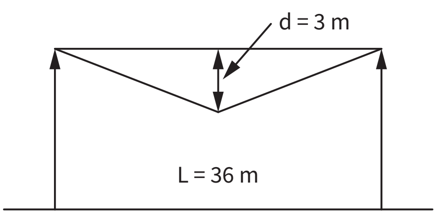
Radial stress a tD = 400 = 0 MPa

1.3 Stress distribution diagram for compound cylinder



Question 2: Tension in cables

2.



2.1 Weight per metre carried by each of the two main cables

Load per metre on bridge $= \frac{\text{Load on bridge}}{\text{Length}} = \frac{3\ 024k}{36} = 84$ kNm

Load per metre per cable $= \frac{\text{Load per metre on bridge}}{2} = \frac{84\text{k}}{2} = 42$ kNm

2.2 Minimum and maximum tension in each cable

$\therefore F_H = F_{min} = \frac{wL^2}{8d} = \frac{42k\tau \times 36^2}{8 \times 3} = 2{,}268$ MN

$\therefore F_{tmax} = \sqrt{F_H^2 + \left(w\frac{L}{2}\right)^2} = \sqrt{(2{,}268\ M)^2 + \left(42k \times \frac{36}{2}\right)^2} = 2{,}391$ MN

2.3 Diameter of the cable with an ultimate stress of material of 320 MPa and a factor of safety of 8

Working safe stress $= \frac{\sigma_{Ult}}{FOS} = \frac{320\ M}{8} = 40$ MPa

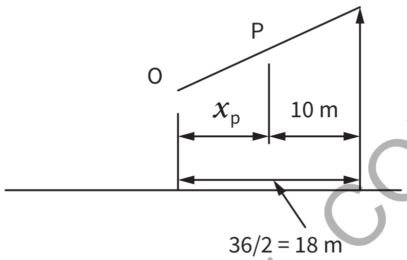
$\therefore \sigma_{safe} = \frac{F_{Tmax}}{A_{cable}}$

$\therefore A_{cable} = \frac{F_{Tmax}}{\sigma_{safe}} = \frac{2{,}391\ M}{40\ M} = 0{,}059775\ m^2$

$\therefore A = \frac{\pi}{4}D^2$

$\therefore D = \sqrt{\frac{0{,}059775 \times 4}{\pi}} = 275{,}877\ mm$

2.4 Tension in cable 10 m from the support measured horizontally



Tension in the cable at point P: From turning point O = x_p = 18 – 10 = 8 m

$\therefore$ Tension at P = $F_{tp} = \sqrt{F_H^2 + (wx_p)^2}$

$\therefore F_{tp} = \sqrt{(2{,}268\ M)^2 + (42k \times 8)^2} = 2{,}293\ MN$

Question 3: Combined bending and twisting of shafts

3. 3.1 $\therefore T_e = \frac{\pi D^3}{16}\tau = \frac{\pi 0{,}08^3}{16} \times 50 \times 10^6 = 5{,}027\ kNm$

$\therefore T_e = \sqrt{M^2 + T^2}$

$\therefore M = \sqrt{5\ 027^2 - 4\ 000^2} = 3{,}045\ kNm$

3.2 Maximum BM principal stress:

$\therefore M_e = \frac{\pi D^3}{32}\sigma = \frac{\pi \times 0{,}08^3}{32} \times 75 \times 10^6 = 3{,}77\ kNm$

$\therefore M_e = 0{,}5\left[M + \sqrt{M^2 + T^2}\right]$

$\therefore 2\ M_e - M = \sqrt{M^2 + T^2}$

$\therefore (2 \times 3\ 770 - M)^2 = M^2 + 4\ 000^2$

$7\ 540^2 - 7\ 540M - 7\ 540M + M^2 = M^2 + 4\ 000^2$

$\therefore 15\ 080M = 7\ 540^2 - 4\ 000^2$

$\therefore M = \frac{40{,}852 \times 10^6}{15\ 080} = 2{,}709\ kNm$

3.3 Maximum bending moment for shaft

$M_{max} = 2{,}709$ kNm

A bending moment of 3,045 kNm will cause a principal stress higher than 75 MPa and the shaft will fail.

3.4 Actual shear stress:

$\therefore T_e = \sqrt{M^2 + T^2} = \sqrt{2\ 709^2 + 4\ 000^2} = 4{,}831$ kNm

$\therefore T_e = \dfrac{\pi D^3}{16}\tau$

$\therefore \tau = \dfrac{16 \times 4\ 831}{\pi \times 0{,}08^3} = 48{,}055$ MPa

Question 4: Bending and deflection of beams

4. 4.1 Dimensions for the pipe:

$\Delta_{max@A} = \Delta_{UDL@A} + \Delta_{pl@A}$

$\therefore 0{,}007 = \left[\dfrac{wa^4}{8EI} + \left(\dfrac{wa^3}{6EI} \times b\right)\right] + \dfrac{WL^3}{3EI}$

$\therefore 0{,}007I = \left[\dfrac{20k \times 1{,}25^4}{8 \times 200G} + \left(\dfrac{20k \times 1{,}25^3}{6 \times 200G} \times 1{,}25\right)\right] + \dfrac{2k \times 2{,}5^3}{3 \times 200G}$

$\therefore 0{,}007I = 3{,}052 \times 10^{-8} + 4{,}069 \times 10^{-8} + 5{,}208 \times 10^{-08}$

$\therefore I = 17{,}613 \times 10^{-6}\,m^4$

$D = 2d$ given

$\therefore I = 17{,}613 \times 10^{-6} = \dfrac{\pi}{64}\left([2d]^4 - d^4\right)$

$\therefore 15d^4 = 3{,}588 \times 10^{-4}$

$\therefore d = \sqrt[4]{2{,}392 \times 10^{-5}} = 69{,}934$ mm

$D = 139{,}869$ mm

4.2 Select the lightest I-section taper flange

$I = 17{,}613 \times 10^{-6}$

$203 \times 102 \times 25{,}3$ kg/m

$I = 22{,}97 \times 10^{-6}\,m^4$

4.3 Maximum bending stress in the selected I-section

$\therefore M_{max} = M_{UDL} + M_{pl} + M_{weight}$

$M_{max} = \dfrac{wa^2}{2} + wL + \dfrac{w_{weight}L^2}{2}$

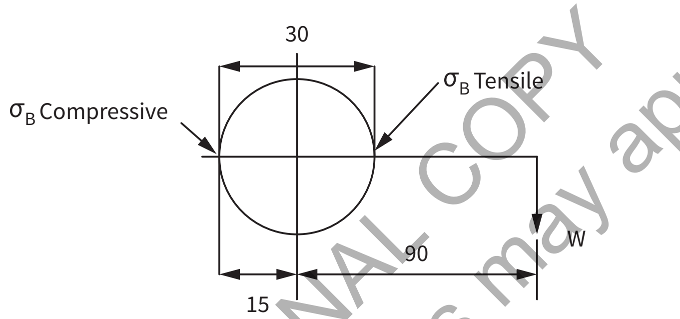
$$M_{max} = \frac{20k \times 1{,}25^2}{2} + 2k \times 2{,}5 + \frac{25{,}3 \times 9{,}81 \times 2{,}5^2}{2}$$

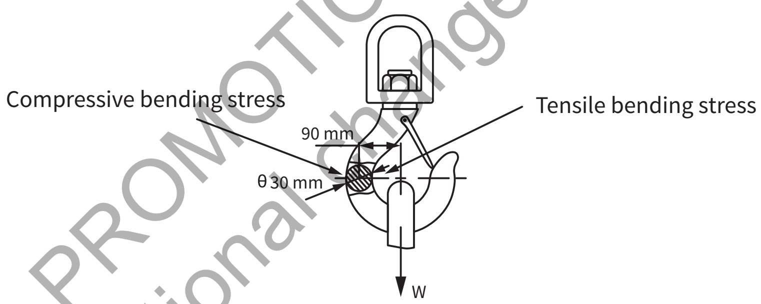
$$M_{max} = 21{,}401 \text{ kNm}$$

$$\therefore \text{Maximum stress} = \frac{My}{I} = \frac{21401 \times 0{,}2032}{22{,}97 \times 10^{-6} \times 2} = 94{,}66 \text{ MPa}$$

Question 5: Combined bending and direct stress

5. 5.1 The mass that can be lifted





$$\text{Direct stress} = \sigma_D = \frac{W}{\frac{\pi}{4}0{,}03^2} = 1\,414{,}711\text{W Pa (Tensile)}$$

$$\text{Bending stress} = \sigma_B = \frac{My}{I} = \frac{(W \times 0{,}09) \times 0{,}015}{\frac{\pi}{64}0{,}03^4 = 33\,953{,}055\text{W}}$$

$$\therefore \sigma_{max} = \sigma_D + \sigma_B = -90\text{M} = -1\,414{,}711\text{W} - 33\,953{,}055\text{W}$$

$$\therefore W = 2{,}545 \text{ kN}$$

$$\therefore \text{Mass} = \frac{2{,}545k}{9{,}81} = 259{,}398 \text{ kg}$$

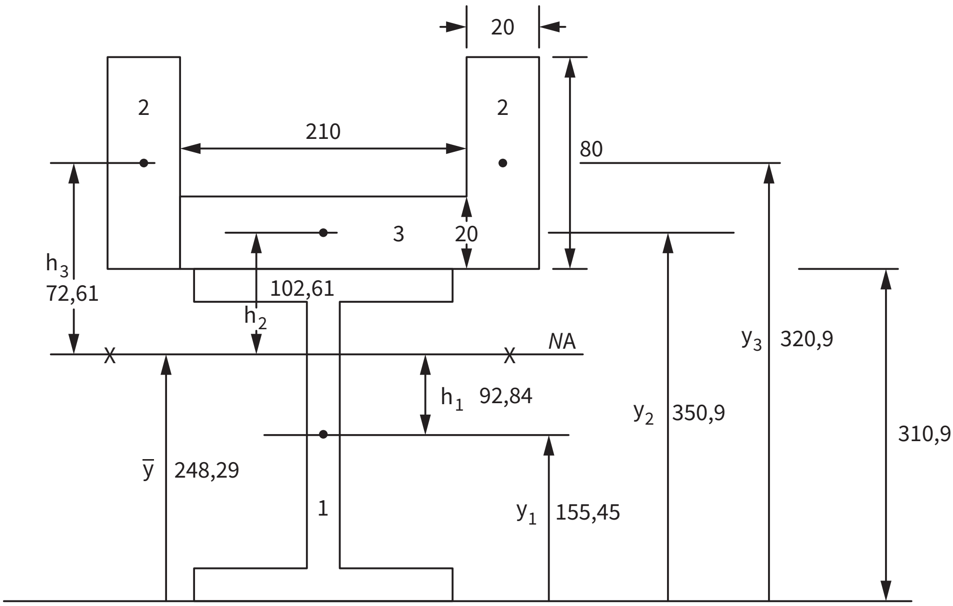
5.2 The minimum stress in the hook, magnitude and nature

$$\sigma_{min} = -\sigma_D + \sigma_B$$

$$\sigma_{min} = -(1\,414{,}711 \times 2{,}545k) + (33\,953{,}055 \times 2{,}545k)$$

$$= 82{,}81 \text{ MPa (Compressive)}$$

Question 6: Shear stress in beams



No	Area	Y-distance	Ay
1	$6{,}821 \times 10^{-3}$	0,15545	$1{,}0603 \times 10^{-3}$
2	$2 \times 0{,}08 \times 0{,}02 = 3{,}2 \times 10^{-3}$	0,3509	$1{,}1229 \times 10^{-3}$
3	$0{,}21 \times 0{,}02 = 4{,}2 \times 10^{-3}$	0,3209	$1{,}3478 \times 10^{-3}$
A total	0,014221	ΣA – moments	$3{,}531 \times 10^{-3}$

$\bar{y}A_T = \Sigma A - \text{moments}$

$\bar{y} = \frac{3{,}531 \times 10^{-3}}{0{,}014221} = 248{,}29 \text{ mm}$

$h_1 = \bar{y} - y_1 = 248{,}29 - 155{,}45 = 92{,}84 \text{ mm}$

$h_2 = y_2 - \bar{y} = 350{,}9 - 248{,}29 = 102{,}61 \text{ mm}$

$h_3 = y_3 - \bar{y} = 320{,}9 - 248{,}29 = 72{,}61 \text{ mm}$

$I_{xx\text{ Total}} = I_1 + I_2 + I_3$

$I_1 = 116{,}9 \times 10^{-6} + \left(6{,}821 \times 10^{-3} \times 0{,}09284^2\right) = 1{,}7569 \times 10^{-4} \text{ m}^4$

$I_2 = 2\left[\frac{0{,}02 \times 0{,}08^3}{12} + \left(3{,}2 \times 10^{-3} \times 0{,}10261^2\right)\right] = 3{,}4546 \times 10^{-5} \text{ m}^4$

$I_3 = \left[\frac{0{,}21 \times 0{,}02^3}{12} + \left(4{,}2 \times 10^{-3} \times 0{,}07261^2\right)\right] = 2{,}2283 \times 10^{-5} \text{ m}^4$

$\therefore I_{xx} = 2{,}3252 \times 10^{-4} \text{ m}^4$

First moment of area $= Q = 2A_2 y'_2 + A_3 y'_3$

$Q = 2 \times 32, \times 10^{-3} \times 0{,}10261 + (4{,}2 \times 10^{-3} \times 0{,}07261) = 9{,}617 \times 10^{-4}\ \text{m}^3$

Shear force at joint $= q = \frac{VQ}{I} = \frac{100k \times 9{,}617 \times 10^{-4}}{2{,}3252 \times 10^{-4}} = 413{,}599$ kN/m

Spacing of bolts over 1 m: $R = A_{bolt} \tau_{bolt} n1$

$R = \frac{\pi \times 0{,}017^2}{4} \times 80M \times n \times 1 = 18{,}158n$ kN

Force in bolt = Force in joint

$R = q \quad \therefore 18\ 158\ n = 413\ 599$

Number of bolt $= n = 22{,}77$ use 23 bolts

Spacing $= S = \frac{\text{Unit length} \times \text{number of rows}}{\text{Number of bolts}} = \frac{1\ 000 \times 2}{23} = 86{,}96$ mm

Question 7: Closed-coiled helical springs

7. 7.1 Wire diameter

Deflection $= \delta = \frac{W}{S} = \frac{45}{900} = 0{,}05$ m

Strain energy $= U = \frac{1}{2} W\delta = 0{,}5 \times 45 \times 0{,}05 = 1{,}125$ J

But $U = \frac{\tau^2 V}{4G} \quad \therefore$ Volume $= \frac{1{,}125 \times 4 \times 40 \times 10^9}{(120 \times 10^6)^2} = 1{,}563 \times 10^{-5}\ \text{m}^3$

Also volume of wire $= \pi D n \times \frac{\pi d^2}{4} = 2{,}467\ Dnd^2 = 1{,}563 \times 10^{-5}$ … ①

Number of coils $= n = \frac{L_{solid}}{d} = \frac{0{,}045}{d}$ … ②

Shear stress wire $= \tau = \frac{8WD}{\pi d^3} \quad \therefore D = \frac{120 \times 10^6 \times \pi \times d^3}{8 \times 45}$

$D = 1{,}047 \times 10^6 d^3$ … ③

Substitute ② and ③ into ①:

$\therefore 2{,}467 \times 1{,}047 \times 10^6 d^3 \times \frac{0{,}045}{d} d^2 = 1{,}563 \times 10^{-5}$

$\therefore d^4 = 1{,}344 \times 10^{-10}$

$\therefore d = 3{,}41$ mm

7.2 Spring diameter

$\therefore D = 1{,}047 \times 10^6 \times 0{,}00341^3 = 41\ 52$ mm

7.3 Number of coils

$n = \frac{45}{3{,}24} = 13{,}89$ coils

Question 8: Transformation of stress

8. 8.1 Normal stress x-face

$$\sigma_{x1} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\sigma_{x1} = \frac{100 + 40}{2} + \frac{100 - 40}{2}\cos - 30 + 0$$

$$\sigma_{x1} = 95{,}98 \text{ MPa}$$

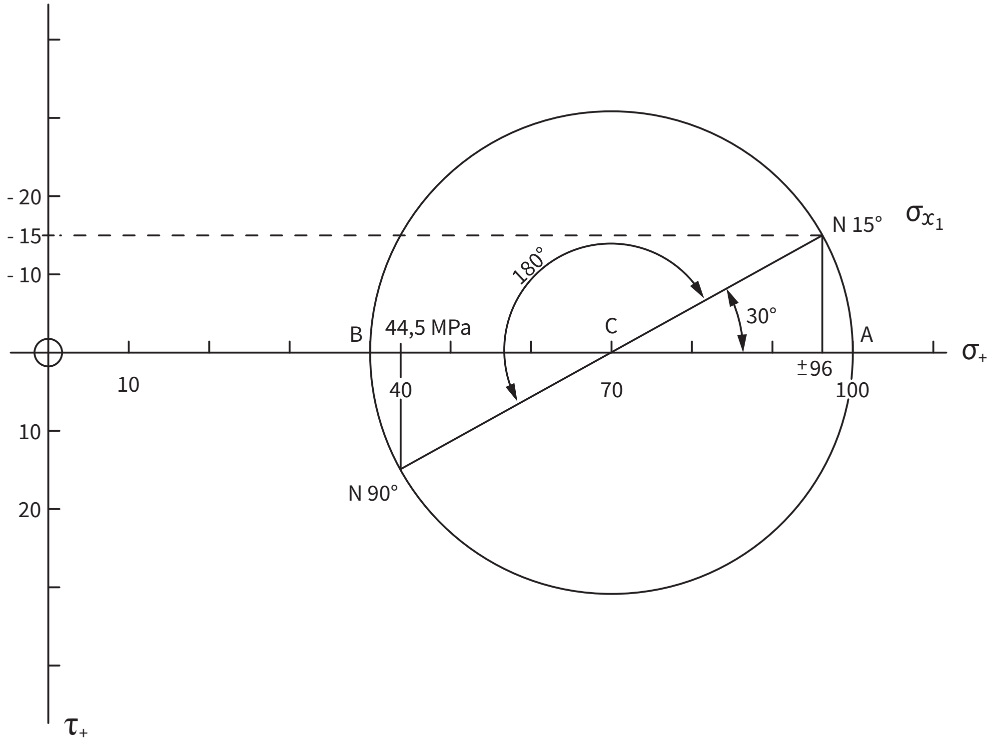
8.2 Shear stress

$$\tau_{xy} = -\left[\frac{\sigma_x - \sigma_y}{2}\right]\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$\tau_{xy} = -\left[\frac{100 - 40}{2}\right]\sin - 30 + 0$$

$$\tau_{xy} = -(-15) = 15 \text{ MPa}$$

8.3 Mohr's circle 15°



$$\sigma_{x1} = \pm 96 \text{ MPa}$$

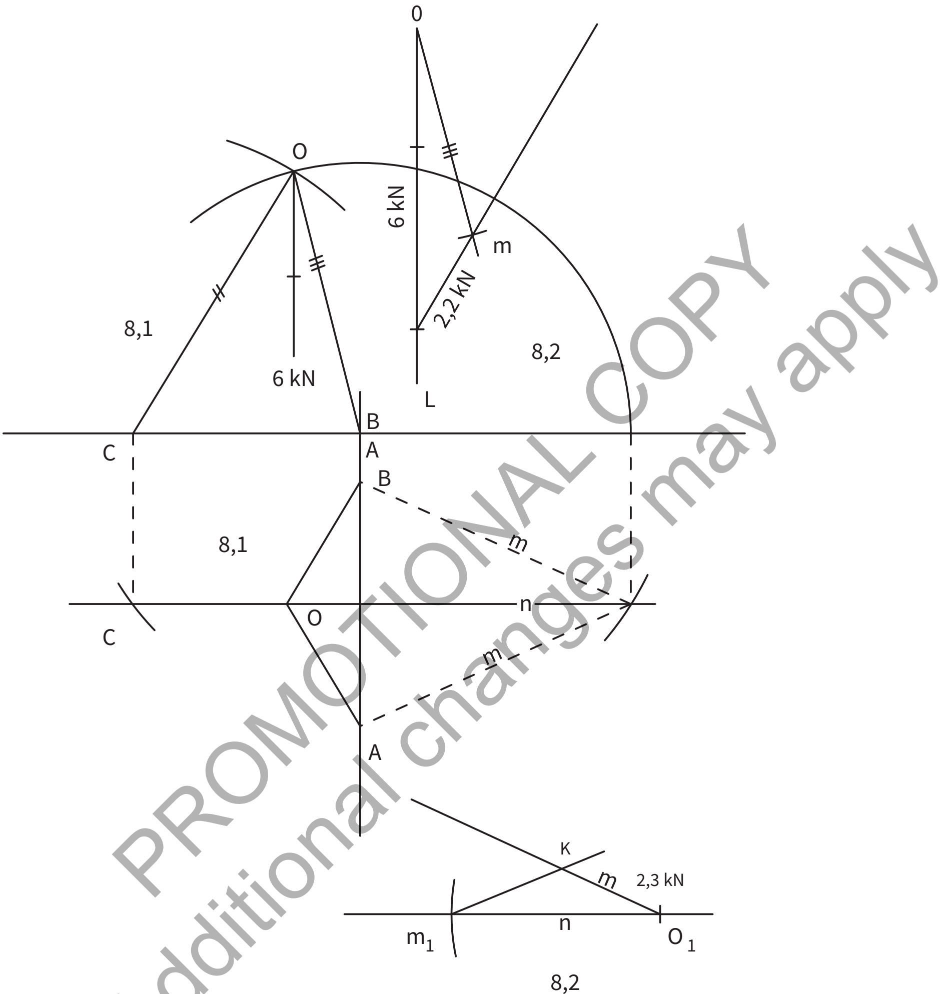
$$\tau_{xy} = 15 \text{ MPa}$$

8.4 90°

$$\sigma_{x1} = \pm 44{,}5 \text{ MPa}$$

Question 9: Forces in structural frameworks

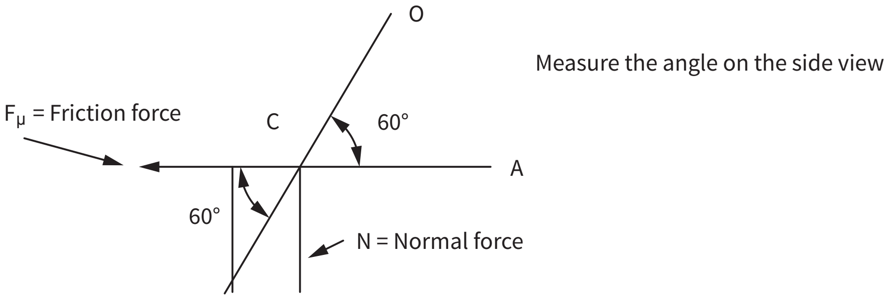
9.



9.1 Tabulated answers

Member	Magnitude
OC	2,2 kN
OA	2,3 kN
OB	2,3 kN

9.2 Coefficient of friction between legs and the ground



From the side view angle ACO = 60° and the force in leg CO = 2,2 kN

$$\therefore \text{Coefficient of friction} = \mu = \frac{F_\mu}{N} = \frac{2\,200 \cos 60°}{2\,200 \sin 60°} = 0{,}577$$