

# Exemplar examination paper

Time: 3 hours

Marks: 100

## INSTRUCTIONS AND INFORMATION

1. Answer all the questions.
2. Read all the questions carefully.
3. Number the answers according to the numbering system used in this question paper.
4. Show all intermediate steps and simplify where possible.
5. All answers must be rounded off to THREE decimals.
6. Questions may be answered in any order, but subsections of questions must be kept together.
7. Sketches must be large, neat and fully labelled.
8. Start each question on a new page.
9. Only use a black or a blue pen.
10. Write neatly and legibly.

## QUESTION 1

1.1 Determine the following limits:

$$1.1.1 \lim_{x \rightarrow 3} \frac{6 - 2x}{\sqrt{x + 22} - \sqrt{10x - 5}} \quad (3)$$

$$1.1.2 \lim_{x \rightarrow -1} \frac{\frac{1}{5 + 4x} + \frac{1}{x}}{2x + 2} \quad (2)$$

1.2 Given:  $\ln y = \lim_{x \rightarrow 4} \frac{\sin(x - 4)}{x - 4}$ , calculate the numerical value of:

$$1.2.1 \ln y \quad (2)$$

$$1.2.2 y \quad (1)$$

1.3 Determine the value(s) of  $x$  for which  $f(x)$  is discontinuous if:

$$f(x) = \frac{\tan 3x}{\cos 2x} \quad (2)$$

[10]

## QUESTION 2

2.1 Given:  $f(x) = -3x^6$

Determine the simplest form of:

$$2.1.1 f(x + h) \quad (2)$$

$$2.1.2 f(x + h) - f(x) \quad (1)$$

$$2.1.3 \frac{f(x + h) - f(x)}{h} \quad (1)$$

$$2.1.4 \lim_{h \rightarrow 0} \frac{f(x + h) - f(x)}{h} \quad (1)$$

- 2.2 Determine  $\frac{dy}{dx}$  in each of the following cases:  
 (Simplification not required)
- 2.2.1  $y = \tan[(7 - x^3)(\ln x)^2]$  (4)
- 2.2.2  $y = \sqrt[3]{\cos(11 - x^2) + \sqrt{\ln x}}$  (4)
- 2.3 Calculate  $\frac{dy}{dx}$  if  $y = \frac{\sin(6x + x^2)}{(7 - x^4)^3}$  with the aid of logarithmic differentiation. (4)
- 2.4 Determine  $\frac{dy}{dx}$  of implicit function  $\cos(x^2 + 3y) + xe^{y^2} = 5$  (5)
- [22]

### QUESTION 3

- 3.1 Given:  $f(x) = 4x^3 - 10x + 3$
- 3.1.1 One root of the equation  $4x^3 - 10x + 3 = 0$  is close to 0,3.  
 Use Taylor's/Newton's method twice to determine a better approximation of this root (root correct to THREE decimals). (4)
- 3.1.2 Determine the coordinate of the point of inflection of  $f(x)$ . (2)
- 3.1.3 Draw up a table of  $x$  and  $f(x)$ , where  $x$  ranges from  $x = -2$  to  $x = 2$ . (2)
- 3.2 Your customer needs an enclosed rectangular box that will hold  $20 \text{ m}^3$ . You are not given any dimensions, but the base must be five times longer than it is wide. You determine that the cost of material for the sides is  $\text{R}4/\text{m}^2$  while the cost of the material for the base is  $\text{R}16/\text{m}^2$ .  
 Calculate the dimensions of the box that will minimise the cost. (6)
- [14]

### QUESTION 4

- 4.1 Determine  $\int y \, dx$  in each of the following cases.
- 4.1.1  $y = \sec^2 3x(7 \tan 3x - \tan^2 3x + 5)$  (4)
- 4.1.2  $y = \frac{3x + 4}{\sqrt{1 - 16x^2}}$  (4)
- 4.1.3  $y = \frac{5x^3 - 23x^2 + 15x + 12}{5x + 2}$  (5)
- 4.1.4  $y = \cos^3 x \sin x$  (3)
- 4.1.5  $y = \tan^3 x$  (6)
- 4.2 Determine  $\int y \, dx$  by resolving the integral into partial fractions:  
 $y = \frac{x + 8}{(x - 3)(x + 1)}$  (5)
- [27]

### QUESTION 5

- 5.1 Evaluate the integral:  
 $\int_2^{10} \left[ \frac{(\ln 2x)^2}{x} + \frac{e^x}{e^x + 2} \right] dx$  (5)

**QUESTION 6**

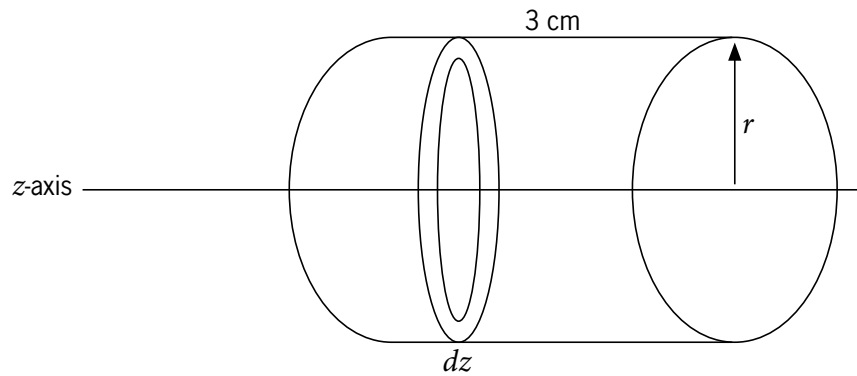
Given:  $y = 4x - 2x^2$  and the  $x$ -axis.

- 6.1 Calculate the coordinates of the points of intersection. (2)
- 6.2 Make a neat sketch to show the enclosed area, the representative strip and the point of intersection. (2)
- 6.3 Calculate the magnitude of the area in QUESTION 6.2. (3)
- 6.4 Calculate the volume of the solid of revolution formed when the area in QUESTION 6.2 is rotated about the  $x$ -axis. (4)

**[11]**

**QUESTION 7**

- 7.1 Calculate the moment of inertia of a flywheel of radius 30 cm and thickness of 3 cm about an axis through its centre and perpendicular to the flywheel. The mass of the flywheel is 15 kg.



**[4]**

**QUESTION 8**

- 8.1 Determine the particular solution of:

$$\frac{dy}{dx} = -\frac{x}{ye^{x^2}} \text{ at } (2; 0) \quad (4)$$

- 8.2 Determine the general solution of:

$$\sec x \cdot \frac{d^2y}{dx^2} = 1 + \tan x + \frac{x^2}{\cos x} \quad (3)$$

**[7]**

**TOTAL: 100**

## Formula sheet

Any applicable formula may also be used.

### Trigonometry

$$\sin^2 x + \cos^2 x = 1 \qquad \cos^2 x = 1 - \sin^2 x \qquad \sin^2 x = 1 - \cos^2 x$$

$$1 + \tan^2 x = \sec^2 x \qquad \tan^2 x = \sec^2 x - 1$$

$$1 + \cot^2 x = \operatorname{cosec}^2 x \qquad \cot^2 x = \operatorname{cosec}^2 x - 1$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

$$\sin^2 A = \frac{1}{2}(1 - \cos 2A)$$

$$\cos^2 A = \frac{1}{2}(1 + \cos 2A)$$

$$\sin A \pm B = \sin A \cos B \pm \cos A \sin B$$

$$\cos A \pm B = \cos A \cos B \mp \sin A \sin B$$

$$\tan A \pm B = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin A \cos B = \frac{1}{2}[\sin(A + B) + \sin(A - B)]$$

$$\cos A \sin B = \frac{1}{2}[\sin(A + B) - \sin(A - B)]$$

$$\cos A \cos B = \frac{1}{2}[\cos(A + B) + \cos(A - B)]$$

$$\sin A \sin B = \frac{1}{2}[\cos(A - B) - \cos(A + B)]$$

$$\tan x = \frac{\sin x}{\cos x}; \sin x = \frac{1}{\operatorname{cosec} x}; \cos x = \frac{1}{\sec x}$$

$$\cot x = \frac{\cos x}{\sin x}; \operatorname{cosec} x = \frac{1}{\sin x}; \sec x = \frac{1}{\cos x}$$

### Binomial theorem

$$(x + h)^n = x^n + nx^{n-1}h + \frac{n(n-1)}{2 \times 1}x^{n-2}h^2 + \frac{n(n-1)(n-2)}{3 \times 2 \times 1}x^{n-3}h^3 + \dots \text{ for } n \in \mathbb{N}$$

### Differentiation

$$e = -\frac{f(a)}{f'(a)}$$

$$r = a + e$$

### Table of differentiation and integration

$f(x)$	$\frac{d}{dx}f(x)$	$\int f(x) dx$
$a$	$0$	$ax + c$
$x^n$	$nx^{n-1}$	$\frac{x^{n+1}}{n+1} + C, \quad (n \neq -1)$
$ax^n$	$a \frac{d}{dx}x^n = nax^{n-1}$	$a \int x^n dx = \frac{ax^{n+1}}{n+1}$
$e^x$	$e^x$	$e^x + c$
$a^x$	$a^x \cdot \ln a$	$\frac{a^x}{\ln a} + C$
$\ln x$	$\frac{1}{x}$	_____
$\log_a x$	$\frac{1}{x \ln a}$	_____
$\sin x$	$\cos x$	$-\cos x + C$
$\cos x$	$-\sin x$	$\sin x + C$
$\tan x$	$\sec^2 x$	$\ln(\sec x) + C$
$\cot x$	$-\operatorname{cosec}^2 x$	$\ln(\sin x) + C$
$\sec x$	$\sec x \tan x$	$\ln(\sec x + \tan x) + C$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cos x$	$\ln(\operatorname{cosec} x + \cot x) + C$
$\sin^{-1} x$	$\frac{1}{\sqrt{1-x^2}}$	_____
$\cos^{-1} x$	$\frac{-1}{\sqrt{1-x^2}}$	_____
$\tan^{-1} x$	$\frac{1}{1+x^2}$	_____
$\cot^{-1} x$	$\frac{-1}{1+x^2}$	_____
$\sec^{-1} x$	$\frac{1}{\sqrt{x^2-1}}$	_____
$\operatorname{cosec}^{-1} x$	$\frac{-1}{x\sqrt{x^2-1}}$	_____
$\frac{1}{\sqrt{a^2-x^2}}$	_____	$\sin^{-1}\left(\frac{x}{a}\right) + C$
$\frac{1}{a^2+x^2}$	_____	$\frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right) + C$
$\frac{1}{x\sqrt{x^2+a^2}}$	_____	$\frac{1}{a} \sec^{-1}\left(\frac{x}{a}\right) + C$
$\sqrt{a^2-x^2}$	_____	$\frac{a^2}{2} \sin^{-1}\left(\frac{x}{a}\right) + \frac{x}{2} \sqrt{a^2-x^2} + C$
$\frac{1}{x^2-a^2}$	_____	$\frac{1}{2a} \ln\left(\frac{x-a}{x+a}\right) + C$
$\frac{1}{a^2-x^2}$	_____	$\frac{1}{2a} \ln\left(\frac{a+x}{a-x}\right) + C$

### Product rule

$$y = u(x) \cdot v(x)$$

$$\frac{dy}{dx} = v \cdot \frac{du}{dx} + u \cdot \frac{dv}{dx}$$

$$= v \cdot u' + u \cdot v'$$

### Quotient rule

$$y = \frac{u(x)}{v(x)}$$

$$\frac{dy}{dx} = \frac{v \cdot \frac{du}{dx} - u \cdot \frac{dv}{dx}}{v^2}$$

$$= \frac{v \cdot u' - u \cdot v'}{v^2}$$

### Chain rule

$$y = f(u(x))$$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

### Integration

$$\int f(x) \cdot g'(x) dx = f(x) \cdot g(x) - \int f'(x) \cdot g(x) dx$$

$$\int [f(x)]^n \cdot f'(x) dx = \frac{[f(x)]^{n+1}}{n+1} + C (n \neq -1)$$

$$\int \frac{f'(x)}{f(x)} dx = \ln f(x) + C$$

$$\frac{f(x)}{(ax+b)(cx+d)} = \frac{A}{ax+b} + \frac{B}{cx+d}$$

$$\frac{f(x)}{(x+a)^n} = \frac{A}{x+a} + \frac{B}{(x+a)^2} + \frac{C}{(x+a)^3} + \dots + \frac{Z}{(x+a)^n}$$

### Applications of integration

#### Areas

$$A_x = \int_a^b y dx; A_x = \int_a^b (y_T - y_B) dx$$

$$A_y = \int_c^d x dy; A_y = \int_c^d (x_R - x_L) dy$$

#### Volumes

$$V_x = \pi \int_a^b y^2 dx; V_x = \pi \int_a^b (y_T^2 - y_B^2) dx$$

$$V_y = \pi \int_c^d x^2 dy; V_y = \pi \int_c^d (x_R^2 - x_L^2) dy$$

#### Second moments of area

$$I_x = \int_a^b r^2 dA; I_y = \int_c^d r^2 dA$$

#### Moments of inertia

Mass = density  $\times$  volume

$$m = \rho V$$

Definition:  $I = mr^2$

$$\text{General: } I = \int_a^b r^2 dm = \rho \int_a^b r^2 dV$$

## Exemplar examination paper memorandum

### QUESTION 1

$$\begin{aligned}
 1.1 \quad 1.1.1 \quad \lim_{x \rightarrow 3} \frac{6 - 2x}{\sqrt{x+22} - \sqrt{10x-5}} &\rightarrow \left(\frac{0}{0}\right) \\
 &= \lim_{x \rightarrow 3} \frac{-2}{\frac{1}{2\sqrt{x+22}} - \frac{10}{2\sqrt{10x-5}}} \checkmark\checkmark \\
 &= \frac{20}{9} \checkmark \qquad (3)
 \end{aligned}$$

$$\begin{aligned}
 1.1.2 \quad \lim_{x \rightarrow -1} \frac{\frac{1}{5+4x} + \frac{1}{x}}{2x+2} &\rightarrow \left(\frac{0}{0}\right) \\
 &= \lim_{x \rightarrow -1} \frac{-\frac{4}{(5+4x)^2} - \frac{1}{x^2}}{2} \checkmark \\
 &= -\frac{5}{2} \checkmark \qquad (2)
 \end{aligned}$$

$$\begin{aligned}
 1.2 \quad 1.2.1 \quad \ln y = \lim_{x \rightarrow 4} \frac{\sin(x-4)}{x-4} &\rightarrow \left(\frac{0}{0}\right) \\
 &= \lim_{x \rightarrow 4} \frac{\cos(x-4)}{1} \checkmark \\
 &= 1 \checkmark \qquad (2)
 \end{aligned}$$

$$1.2.2 \quad y = e^1 = e \checkmark \qquad (1)$$

$$\begin{aligned}
 1.3 \quad f(x) \text{ is discontinuous if } \cos 2x &= 0 \\
 \cos 2x &= 0 \\
 2x &= 90^\circ \\
 \therefore x &= 45^\circ \checkmark \quad \text{or} \quad x = 135^\circ \checkmark \qquad (2)
 \end{aligned}$$

[10]

### QUESTION 2

$$2.1 \quad f(x) = -3x^6$$

$$\begin{aligned}
 2.1.1 \quad f(x+h) &= -3(x+h)^6 \\
 &= -3 \left[ \frac{x^6 h^0}{0!} + \frac{6x^5 h}{1!} + \frac{30x^4 h^2}{2!} + \frac{120x^3 h^3}{3!} + \dots \right] \\
 &= -3x^6 - 18x^5 h - 45x^4 h^2 - 90x^3 h^3 + \dots \checkmark\checkmark \qquad (2)
 \end{aligned}$$

$$\begin{aligned}
 2.1.2 \quad f(x+h) - f(x) \\
 &= -18x^5 h - 45x^4 h^2 - 90x^3 h^3 + \dots \checkmark \qquad (1)
 \end{aligned}$$

$$2.1.3 \quad \frac{f(x+h) - f(x)}{h} = -18x^5 - 45x^4 h - 90x^3 h^2 + \dots \checkmark \qquad (1)$$

$$2.1.4 \quad \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = -18x^5 \checkmark \qquad (1)$$

2.2 Determine  $\frac{dy}{dx}$  in each of the following cases:

2.2.1  $y = \tan[(7 - x^3)(\ln x)^2]$

$$\begin{aligned} \frac{dy}{dx} &= \sec^2[(7 - x^3)(\ln x)^2] \times \left[-3x(\ln x)^2 + (7 - x^3) \times 2 \ln x \times \frac{1}{x}\right] \checkmark\checkmark\checkmark \\ &= \sec^2[(7 - x^3)(\ln x)^2] \times \left[-3x(\ln x)^2 + \frac{2 \ln x(7 - x^3)}{x}\right] \checkmark \end{aligned} \quad (4)$$

2.2.2  $y = \sqrt[3]{\cos(11 - x^2) + \sqrt{\ln x}}$

$$\begin{aligned} \frac{dy}{dx} &= \frac{1}{3} [\cos(11 - x^2) + \sqrt{\ln x}]^{-\frac{3}{4}} \times \left[-\sin(11 - x^2) \times 2x + \frac{1}{2}(\ln x)^{-\frac{1}{2}} \times \frac{1}{x}\right] \checkmark\checkmark\checkmark \\ &= \frac{1}{3} [\cos(11 - x^2) + \sqrt{\ln x}]^{-\frac{3}{4}} \times \left[-2x \sin(11 - x^2) \times 2x + \frac{1}{2x\sqrt{\ln x}}\right] \checkmark \end{aligned} \quad (4)$$

2.3  $y = \frac{\sin(6x + x^2)}{(7 - x^4)^3}$

$\ln y = \ln \sin(6x + x^2) - 3 \ln(7 - x^4)$

$$\frac{1}{y} \frac{dy}{dx} = \frac{1}{\sin(6x + x^2)} \times \cos(6x + x^2) \times (6 + 2x) - \frac{3}{(7 - x^4)} \times (-4x^3) \checkmark\checkmark\checkmark$$

$$\frac{dy}{dx} = y \left[ \frac{(6 + 2x)\cos(6x + x^2)}{\sin(6x + x^2)} + \frac{12x^3}{7 - x^4} \right]$$

$$\frac{dy}{dx} = \frac{\sin(6x + x^2)}{(7 - x^4)^3} \left[ \frac{(6 + 2x)\cos(6x + x^2)}{\sin(6x + x^2)} + \frac{12x^3}{7 - x^4} \right] \checkmark \quad (4)$$

2.4  $\cos(x^2 + 3y) + xe^{y^2} = 5$

$$-\sin(x^2 + 3y) \times \left[2x + 3\frac{dy}{dx}\right] + e^{y^2} + 2xye^{y^2} \frac{dy}{dx} = 0 \checkmark\checkmark$$

$$-2x \sin(x^2 + 3y) - 3 \sin(x^2 + 3y) \frac{dy}{dx} + e^{y^2} + 2xye^{y^2} \frac{dy}{dx} = 0 \checkmark$$

$$\frac{dy}{dx} [2xye^{y^2} - 3 \sin(x^2 + 3y)] = 2x \sin(x^2 + 3y) - e^{y^2} \checkmark$$

$$\frac{dy}{dx} = \frac{2x \sin(x^2 + 3y) - e^{y^2}}{2xye^{y^2} - 3 \sin(x^2 + 3y)} \checkmark \quad (5)$$

[22]

### QUESTION 3

3.1 Given:  $f(x) = 4x^3 - 10x + 3$

3.1.1 Let  $x_0 = 0,3$

$$f(0,3) = 0,108$$

$$f'(0,3) = -8,92 \checkmark$$

$$x_1 = 0,3 - \frac{0,108}{-8,92} \checkmark$$

$$= 0,312$$



Now use  $x_1 = 0,312$  ✓

$$f(0,312) = 0,001$$

$$f'(0,312) = -8,831$$

$$x_2 = 0,312 - \frac{0,001}{-8,831}$$

$$= 0,312 \text{ ✓}$$

(4)

3.1.2  $f(x) = 4x^3 - 10x + 3$

$$f'(x) = 12x^2 - 10$$

$$f''(x) = 24x$$

$$24x = 0 \rightarrow x = 0 \text{ ✓}$$

$$f(0) = 3$$

The coordinate of a point of inflection is  $(0; 3)$  ✓

(2)

3.1.3

$x$	-2	-1	0	1	2
$f(x)$	-9	9	3	-3	15

(2)

3.2 Volume = length  $\times$  width  $\times$  height

$$= 5w \times w \times h = 5w^2h$$

Required volume:  $20 = 5w^2h$

$$\therefore h = \frac{20}{5w^2} = \frac{4}{w^2} \text{ ✓}$$

Cost =  $4(2lh + 2wh) + 16(2lw)$  ✓

$$= 4(10wh + 2wh) + 16(10w^2)$$

$$= 48wh + 160w^2$$

$$= \left(48w \times \frac{4}{w^2}\right) + 160w^2$$

$$\therefore C = \frac{192}{w} + 160w^2 \text{ ✓}$$

$$C' = -\frac{192}{w^2} + 320w \text{ ✓}$$

Cost at a minimum where  $C' = 0$ .

$$-\frac{192}{w^2} + 320w = 0 \text{ ✓}$$

$$320w^3 = 192$$

$$w^3 = \frac{192}{320}$$

$$\therefore w = 0,843; l = 4,217; h = 5,629 \text{ ✓}$$

(6)

[14]

**QUESTION 4**

4.1 Determine  $\int y \, dx$  in each of the following cases.

4.1.1  $y = \int \sec^2 3x (7 \tan 3x - \tan^2 3x + 5) \, dx$

Let  $u = \tan 3x$ ;  $\frac{du}{dx} = 3 \sec^2 3x$ ;  $\frac{du}{3} = \sec^2 3x \, dx \checkmark$

$$y = \frac{1}{3} \int 7u - u^2 + 5 \, du$$

$$= \frac{1}{3} \left( \frac{7}{2} u^2 - \frac{1}{3} u^3 + 5u \right) + C$$

$$= \frac{7}{6} \tan^2 3x - \frac{1}{9} \tan^3 3x + \frac{5}{3} \tan 3x + C \checkmark \checkmark \checkmark \quad (4)$$

4.1.2  $y = \int \frac{3x+4}{\sqrt{1-16x^2}} \, dx$

$$= \int \frac{3x}{\sqrt{1-16x^2}} \, dx + \int \frac{4}{\sqrt{1-(4x)^2}} \, dx$$

Let  $u = 1 - 16x^2$  and  $v = 4x$ ;

$$\frac{du}{dx} = -32x; \frac{du}{-32} = x \, dx \checkmark$$

$$\frac{dv}{dx} = 4; \frac{dv}{4} = dx$$

$$y = \frac{-3}{32} \int u^{-\frac{1}{2}} \, du + \frac{4}{4} \int \frac{1}{\sqrt{1-v^2}} \, dv \checkmark$$

$$= \frac{-3}{16} u^{\frac{1}{2}} + \frac{4}{4} \sin^{-1} v + C$$

$$= \frac{-3}{16} (1 - 16x^2)^{\frac{1}{2}} + \sin^{-1} 4x + C \checkmark \checkmark \quad (4)$$

4.1.3  $y = \int \frac{5x^3 - 23x^2 + 15x + 12}{5x+2} \, dx$

$$= \int x^2 - 5x + 5 + \frac{2}{5x+2} \, dx$$

$$= \frac{x^3}{3} - \frac{5}{2} x^2 + 5x + \frac{2}{5} \ln(5x+2) + C \checkmark \checkmark \checkmark \quad (5)$$

4.1.4  $y = \int \cos^3 x \sin x \, dx$

Let  $u = \cos x$

$$du = -\sin x \, dx \checkmark$$

$$y = -\int u^3 \, du \checkmark$$

$$= -\frac{1}{4} u^4 + C$$

$$= -\frac{1}{4} \cos^4 x + C \checkmark \quad (3)$$

4.1.5  $y = \int \tan^3 x \, dx$

$$\int \tan^3 x \, dx = \int \tan^2 x \tan x \, dx$$

$$= \int (\sec^2 x - 1) \tan x \, dx$$

$$= \int \sec^2 x \tan x \, dx - \int \tan x \, dx \checkmark$$

$$= \int \sec^2 x \tan x \, dx - \ln |\sec x| + C \checkmark$$

Let  $u = \tan x$ ;  $du = \sec^2 x \, dx$  ✓

Then  $\int \sec^2 x \tan x \, dx = \int u \, du = \frac{u^2}{2} + C_1$  ✓

$\therefore y = \frac{u^2}{2} - \ln \sec x + C + C_1$

$= \frac{\tan^2 x}{2} - \ln \sec x + K$  ✓✓ (6)

4.2 Determine  $\int y \, dx$  by resolving the integral into partial fractions:

$$y = \int \frac{x+8}{(x-3)(x+1)} dx$$

$$\frac{x+8}{(x-3)(x+1)} = \frac{A}{x-3} + \frac{B}{x+1}$$

$$x+8 = A(x+1) + B(x-3)$$
 ✓

$$x+8 = Ax + A + Bx - 3B$$

$\therefore A = 2$ ;  $B = -1$  ✓✓

$$y = \int \frac{2}{x-3} - \frac{1}{x+1} dx$$

$$= 2 \ln(x-3) - \ln(x+1) + C$$
 ✓✓ (5)

[27]

**QUESTION 5**

5.1  $\int_2^{10} \left[ \frac{(\ln 2x)^2}{x} + \frac{e^x}{e^x+2} \right] dx$

$$= \int_2^{10} \frac{(\ln 2x)^2}{x} dx + \int_2^{10} \frac{e^x}{e^x+2} dx$$

Let  $u = \ln 2x \, dx$ ;  $du = \frac{1}{x} dx$

$z = e^x + 2$ ;  $dz = e^x dx$  ✓

$$= \int u^2 du + \int \frac{1}{z} dz$$
 ✓

$$= \left[ \frac{(\ln 2x)^3}{3} + \ln(e^x + 2) \right]_2^{10}$$
 ✓

$$= \left[ \frac{(\ln 20)^3}{3} + \ln(e^{10} + 2) \right] - \left[ \frac{(\ln 4)^3}{3} + \ln(e^2 + 2) \right]$$
 ✓

$$= 15,834$$
 ✓ [5]

**QUESTION 6**

6.1  $4x - 2x^2 = 0$

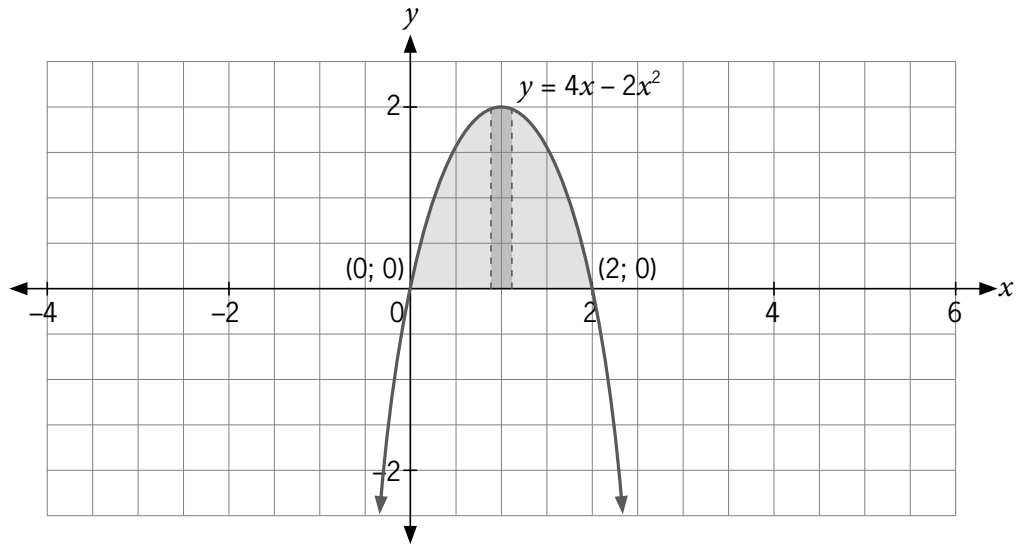
$$2x(2 - x) = 0$$

$\therefore x = 0$  or  $x = 2$

and  $y = 0$

The co-ordinates are  $(0; 0)$  and  $(2; 0)$  ✓✓ (2)

6.2



(2)

$$6.3 \quad A = \int_a^b y \, dx$$

$$A = \int_0^2 4x - 2x^2 \, dx \checkmark$$

$$= \left[ 2x^2 - \frac{2}{3}x^3 \right]_0^2 \checkmark$$

$$= \left[ 2(2)^2 - \frac{2}{3}(2)^3 \right] - \left[ 2(0)^2 - \frac{2}{3}(0)^3 \right]$$

$$= 8 - \frac{16}{3}$$

$$= 2\frac{2}{3} = 2,667 \text{ units}^2 \checkmark$$

(3)

$$6.4 \quad V = \pi \int_0^2 (4x - 2x^2)^2 \, dx$$

$$= \pi \int_0^2 16x^2 - 16x^3 + 4x^4 \, dx \checkmark \checkmark$$

$$= \pi \left[ 16\frac{x^3}{3} - 4x^4 + 4\frac{x^5}{5} \right]_0^2 \checkmark$$

$$= \pi \left[ \left[ 16\frac{2^3}{3} - 4(2)^4 + 4\frac{2^5}{5} \right] - [0] \right]$$

$$= \frac{64}{15} \pi = 13,404 \text{ units}^3 \checkmark$$

(4)

[11]

### QUESTION 7

$$7.1 \quad \text{Moment of inertia is: } \frac{1}{2} r^2 \, dm$$

$$dl_z = \frac{1}{2} r^2 \, dm \checkmark$$

$$I_z = \frac{1}{2} r^2 \int_0^{0,03} dm \checkmark$$

$$= \frac{1}{2} (0,03)^2 (15) \checkmark$$

$$= 0,007 \text{ kg.m}^2 \checkmark$$

[4]

**QUESTION 8**

$$8.1 \quad \frac{dy}{dx} = -\frac{x}{ye^{x^2}}$$

$$y \, dy = -xe^{-x^2} \, dx$$

$$\frac{y^2}{2} = \frac{1}{2}e^{-x^2} + C \checkmark\checkmark$$

$$\frac{2^2}{2} = \frac{1}{2}e^{-0^2} + C$$

$$\therefore C = 1\frac{1}{2} \checkmark$$

$$\frac{y^2}{2} = \frac{1}{2}e^{-x^2} + 1\frac{1}{2} \checkmark$$

(4)

$$8.2 \quad \sec x \cdot \frac{d^2y}{dx^2} = 1 + \tan x + \frac{x^2}{\cos x}$$

$$\frac{d^2y}{dx^2} = \cos x + \sin x + x^2 \checkmark$$

$$\frac{dy}{dx} = -\sin x + \cos x + \frac{x^3}{3} + C_1 \checkmark$$

$$y = -\cos x - \sin x + \frac{x^4}{12} + C_1x + C_2 \checkmark$$

(3)

[7]

**TOTAL: 100**