higher education & training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

NATIONAL CERTIFICATE

MATHEMATICS N5

(16030175)

21 November 2024 (X-paper)
09:00–12:00

Scientific calculators and drawing instruments may be used.

This question paper consists of 6 pages and a formula sheet of 5 pages.

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DEPARTMENT OF HIGHER EDUCATION AND TRAINING
REPUBLIC OF SOUTH AFRICA

NATIONAL CERTIFICATE
MATHEMATICS N5
TIME: 3 HOURS
MARKS: 100

INSTRUCTIONS AND INFORMATION

1. Answer all the questions.
2. Read all the questions carefully.
3. Number the answers according to the numbering system used in this question paper.
4. Show all intermediate steps and simplify where possible.
5. All final answers must be rounded off to THREE decimal places.
6. Questions may be answered in any order, but subsections of questions must be kept together.
7. Sketches must be large, neat and fully labelled
8. Use only a black or blue pen.
9.. Write neatly and legibly.

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QUESTION 1

1.1 Determine the following limits:

1.1.1 $\lim\limits_{x\to\infty} \dfrac{\ln(1+e^x)}{5x}$ (3)

1.1.2 $\lim\limits_{x\to 0} \dfrac{3^x - 2^x}{x^2 - x}$ (2)

1.2 Determine the value(s) of x for which $f(x)$ is continuous if

$$f(x) = \frac{3x + \sin x}{\sqrt{2-x}}$$ (1)

[6]

QUESTION 2

2.1 Determine $\frac{dy}{dx}$ in each of the following cases:

(Simplification NOT required)

2.1.1 $y = \cot^4 \sqrt{\tan 3x^2}$ (5)

2.1.2 $y = \tan\left(\sqrt{\dfrac{1-e^{2x}}{1+e^{2x}}}\right)$ (5)

2.2 Calculate $\frac{dy}{dx}$ if $y = \left(\frac{1}{x}\right)^{3x^2+2}$ with the aid of logarithmic differentiation. (5)

2.3 $x = 3 + \sqrt{x^2 + y^2}$

2.3.1 Determine $\frac{dy}{dx}$ (4)

2.3.2 Determine the equation of the tangent to the graph at point $(4, 3)$. (3)

[22]

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QUESTION 3

3.1 Given:

$$f(x) = 2x^3 - 3x - 2$$

3.1.1 Determine the coordinates of the turning points of $f(x)$. (3)

3.1.2 Draw a table of x and $f(x)$, where x ranges from $x = -2$ to $x = 2$. (2)

3.1.3 Draw a neat graph of $f(x)$ between these values to show the turning points on the graph. (2)

3.1.4 One root of the equation $2x^3 - 3x - 2 = 0$ is close to 1,4. Use Taylor's/ Newton's method TWICE to determine a better approximation of this root (root correct to THREE decimal figures). (4)

3.2 Find the dimensions of a rectangle with a perimeter of 300 m with an area as large as possible. (5)

3.3 A position of a particle is given by the equation

$$s(t) = e^{-t} \ln 3t$$

where $s(t)$ is measured in metres and t in seconds.

3.3.1 Obtain an expression for velocity. (1)

3.3.2 What is the acceleration of the particle at $t = 2s$? (3)

[20]

QUESTION 4

4.1 Determine $\int y\,dx$ in each of the following cases:

4.1.1 $y = 2\sec^2\left(\frac{\pi}{3}x\right)\tan\left(\frac{\pi}{3}x\right)$ (3)

4.1.2 $y = \sin(-6x)\sin(3x)$ (3)

4.1.3 $y = \sin^2\left(\frac{3}{4}x\right)$ (3)

4.1.4 $y = \frac{4x^3 - 6x^2 - 10x + 9}{2x - 1}$ (5)

4.1.5 $y = \cos(-x)\,e^{-4x}$ (5)

4.2 Determine $\int y\,dx$ by resolving the integral into partial fractions:

$$y = \frac{x - 4}{2x^2 - 5x - 3}$$ (5)

[24]

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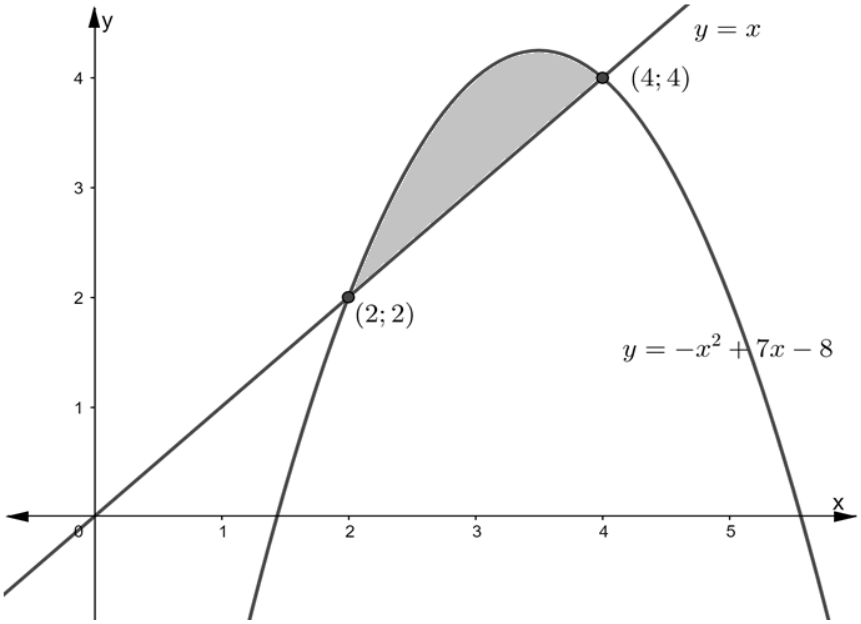
QUESTION 5

5.1 If $f(8) = 0$ and $\int_0^8 f(x)\,dx = -12$, then evaluate $\int_0^8 xf'(x)\,dx$ (4)

5.2 Evaluate the following integral:

$$\int_0^{\infty} 4^{-x}\,dx \quad (3)$$

5.3 Given below is the graph of the functions: $y = -x^2 + 7x - 8$ and $y = x$.



5.3.1 Calculate the magnitude of the area bounded by the graphs. (3)

5.3.2 Calculate the volume of the solid of revolution formed when the area in QUESTION 5.3.1 is rotated about the x-axis. (4)

5.4 Calculate the second moment of area of a circular lamina with radius 15 cm and about an axis through its centre and perpendicular to the plane of the lamina. (6)

[20]

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QUESTION 6

6.1 Determine the particular solution of:

$$\frac{dy}{dx} = \frac{2x + 2xy^2}{y} \qquad y(0) = -2$$ (5)

6.2 Determine the particular solution of:

$$\frac{d^2y}{dx^2} = (x + 2)^2 - 3$$ (3)

[8]

TOTAL: 100

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MATHEMATICS N5

FORMULA SHEET

Any applicable formula may also be used.

TRIGONOMETRY

$\sin^2 x + \cos^2 x = 1$

$1 + \tan^2 x = \sec^2 x$

$1 + \cot^2 x = \text{cosec}^2 x$

$\sin 2A = 2 \sin A \cos A$

$\cos 2A = \cos^2 A - \sin^2 A$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

$\sin^2 A = ½ - ½ \cos 2A$

$\cos^2 A = ½ + ½ \cos 2A$

$\sin (A \pm B) = \sin A \cos B \pm \sin B \cos A$

$\cos (A \pm B) = \cos A \cos B \mp \sin A \sin B$

$$\tan (A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$\sin A \cos B = ½ [\sin (A + B) + \sin (A - B)]$

$\cos A \sin B = ½ [\sin (A + B) - \sin (A - B)]$

$\cos A \cos B = ½ [\cos (A + B) + \cos (A - B)]$

$\sin A \sin B = ½ [\cos (A - B) - \cos (A + B)]$

$$\tan x = \frac{\sin x}{\cos x} ; \sin x = \frac{1}{\text{cosec } x} ; \cos x = \frac{1}{\sec x}$$

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BINOMIAL THEOREM

$$(x+h)^n = x^n + nx^{n-1}\,h + \frac{n(n-1)}{2!}x^{n-2}\,h^2 + \ldots$$

DIFFERENTIATION

$$e = -\frac{f(a)}{f'(a)}$$

$$r = a + e$$

PRODUCT RULE

$$y = u(x)\,.\,v(x)$$

$$\frac{dy}{dx} = u\,.\,\frac{dv}{dx} + v\,.\,\frac{du}{dx}$$

$$= u\cdot v' + v\cdot u'$$

QUOTIENT RULE

$$y = \frac{u(x)}{v(x)}$$

$$\frac{dy}{dx} = \frac{v\,.\,\frac{du}{dx} - u\,.\,\frac{dv}{dx}}{v^2}$$

$$= \frac{v\cdot u' - u\cdot v'}{v^2}$$

CHAIN RULE

$$y = f(u(x))$$

$$\frac{dy}{dx} = \frac{dy}{du}\cdot\frac{du}{dx}$$

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$f(x)$	$\frac{d}{dx} f(x)$	$\int f(x)dx$
ax^n	nax^{n-1}	$\frac{ax^{n+1}}{n+1} + c$
a	0	$ax + c$
e^x	e^x	$e^x + c$
a^x	$a^x.\ln a$	$\frac{a^x}{\ln a} + c$
$\log_e x$	$\frac{1}{x}$	—
$\log_a x$	$\frac{1}{x \ln a}$	—
$\sin x$	$\cos x$	$-\cos x + c$
$\cos x$	$-\sin x$	$\sin x + c$
$\tan x$	$\sec^2 x$	$\ln(\sec x) + c$
$\cot x$	$-\text{cosec}^2 x$	$\ln(\sin x) + c$
$\sec x$	$\sec x \cdot \tan x$	$\ln[\sec x + \tan x] + c$
$\text{cosec } x$	$-\text{cosec } x \cdot \cot x$	$\ln(\text{cosec } x - \cot x) + c$
$\sin^{-1} x$	$\frac{1}{\sqrt{1-x^2}}$	—
$\cos^{-1} x$	$\frac{-1}{\sqrt{1-x^2}}$	—
$\tan^{-1} x$	$\frac{1}{1+x^2}$	—
$\cot^{-1} x$	$\frac{-1}{1+x^2}$	—
$\sec^{-1} x$	$\frac{1}{x\sqrt{x^2-1}}$	—
$\text{cosec}^{-1} x$	$\frac{-1}{x\sqrt{x^2-1}}$	—

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$f(x)$	$\frac{d}{dx} f(x)$	$\int f(x)dx$
$\frac{1}{\sqrt{a^2 - x^2}}$	—	$\sin^{-1}\left(\frac{x}{a}\right) + c$
$\frac{1}{a^2 + x^2}$	—	$\frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + c$
$\frac{1}{x\sqrt{x^2 - a^2}}$	—	$\frac{1}{a}\sec^{-1}\left(\frac{x}{a}\right) + c$
$\sqrt{a^2 - x^2}$	—	$\frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right) + \frac{x}{2}\sqrt{a^2 - x^2} + c$
$\frac{1}{x^2 - a^2}$	—	$\frac{1}{2a} ln\left(\frac{x - a}{x + a}\right) + c$
$\frac{1}{a^2 - x^2}$	—	$\frac{1}{2a} ln\left(\frac{a + x}{a - x}\right) + c$

INTEGRATION

$\int f(x)\, g'(x)\, dx = f(x)\, g(x) - \int f'(x)\, g(x)\, dx$

$\int \frac{f'(x)}{f(x)} dx = \ln f(x) + c$

$\int [f(x)]^n f'(x)\, dx = \frac{[f(x)]^{n+1}}{n+1} + c$

$\frac{f(x)}{(ax + b)(cx + d)} = \frac{A}{ax + b} + \frac{B}{cx + d}$

$\frac{f(x)}{(x + a)^n} = \frac{A}{(x + a)} + \frac{B}{(x + a)^2} + \frac{C}{(x + a)^3} + \cdots \frac{Z}{(x + a)^n}$

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APPLICATIONS OF INTEGRATION

AREAS

$$A_x = \int_a^b y\,dx\,;\; A_x = \int_a^b (y_1 - y_2)\,dx$$

$$A_y = \int_a^b x\,dy\,;\; A_y = \int_a^b (x_1 - x_2)\,dy$$

VOLUMES

$$V_x = \pi\int_a^b y^2\,dx\,;\; V_x = \pi\int_a^b \left(y_1^2 - y_2^2\right)dx$$

$$V_y = \pi\int_a^b x^2\,dy\,;\; V_y = \pi\int_a^b \left(x_1^2 - x_2^2\right)dy$$

SECOND MOMENT OF AREA

$$I_x = \int_a^b r^2\,dA\,;\; I_y = \int_a^b r^2\,dA$$

MOMENTS OF INERTIA

Mass = density × volume

$M = \rho V$

DEFINITION: $I = m\,r^2$

GENERAL: $I = \int_a^b r^2\,dm = \rho\int_a^b r^2\,dV$

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